BIOMATHEMATICS

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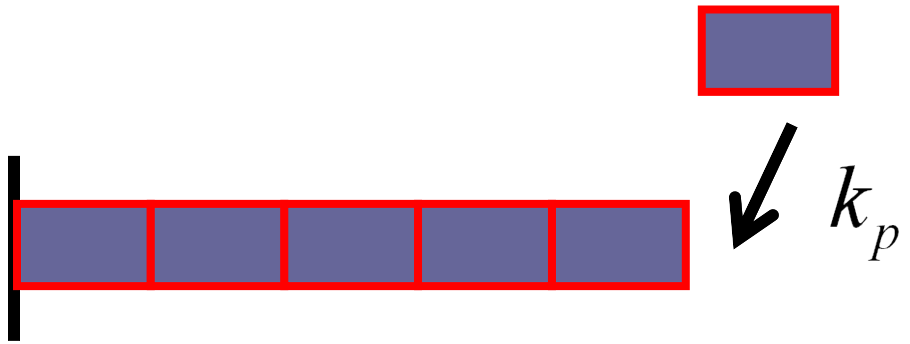
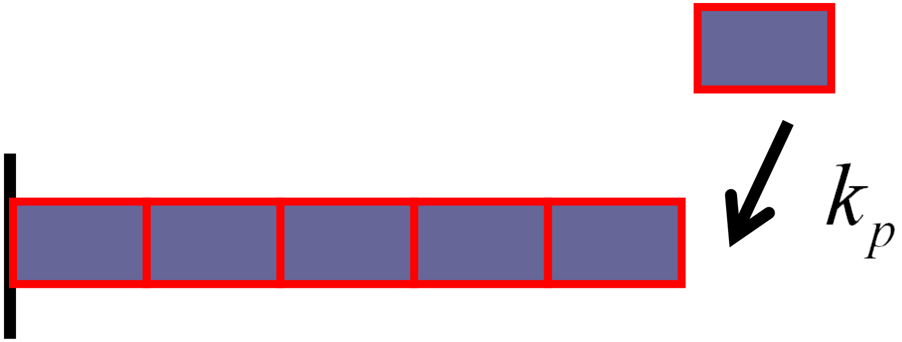
Differential equations

Polymerization of actin



Rate of polymerization

Actin: Length changes with time

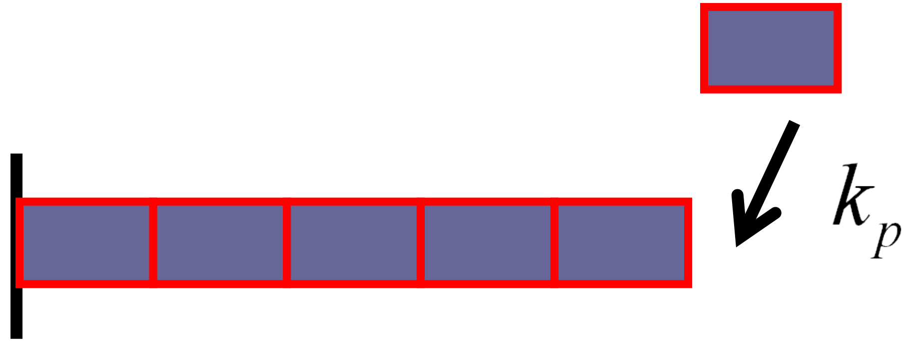


Rate of polymerization

How do we describe this using mathematics ?

Mathematical description

$$\frac{dl}{dt} = k_p$$



Differential equation

Similar: Kinesin moving with constant velocity

Differential equations plays a prominent role in all fields – Physics, Engineering, Economics, Biology…

Integral as “anti-derivative”

Slope $$\frac{dy}{dx} = m$$

Where m is a constant.

We can “integrate” this equation and get y(x)

$$\frac{dy}{dx} = m$$

$$dy = m\,dx$$

$$\int dy = \int m\,dx$$

$$y = mx + c$$

Where ‘c’ is an arbitrary constant

$$\frac{dl}{dt} = k_p$$

$$dl = k_p dt$$

$$\int dl = \int k_p dt$$

$$l = k_p t + c$$

Where ‘c’ is an arbitrary constant

We get length as a function of time

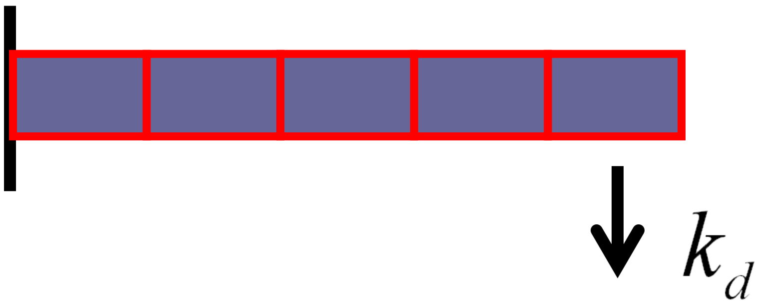
$$l(t) = k_p t + c$$

How do we get "c" ?

Let us take, t=0: $$l(t=0) = c$$

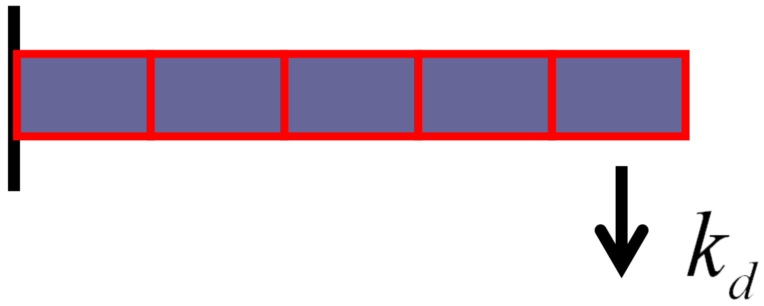
"C" is nothing but the initial length of the actin filament

Depolymerization of actin



Rate of depolymerization

Depolymerization of actin

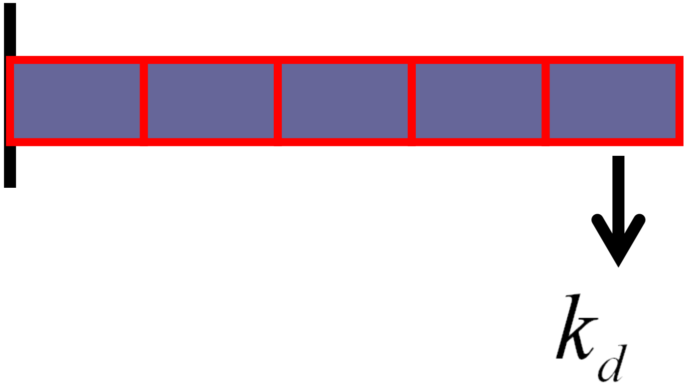


Rate of depolymerization

Length decreases with time

Mathematical description

$$\frac{dl}{dt} = -k_d$$



$$\frac{dl}{dt} = -k_d$$

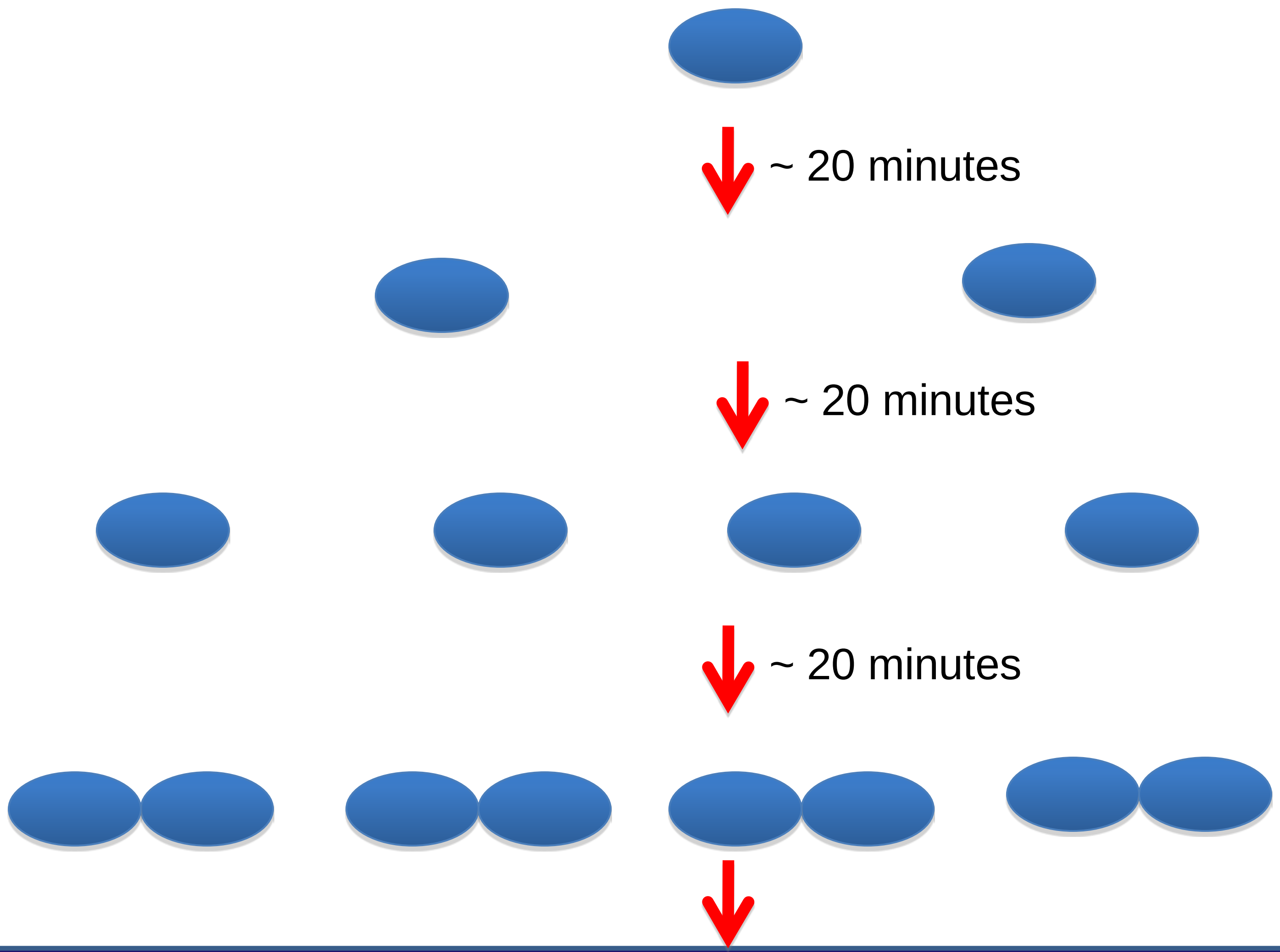
$$dl = -k_d dt$$

$$\int dl = -\int k_d dt$$

$$l = -k_d t + c$$

We get length as a function of time
C=Initial length (length at t=0)

Bacterial growth (eg. E. Coli)



Here, two points to note:

1) Number of bacteria increases with time

2) The more the bacteria, the more the increase

Mathematical description

$$\frac{dN}{dt} = +kN$$

$$\frac{dN}{dt} = kN$$

$$\frac{dN}{N} = kdt$$

$$\int \frac{dN}{N} = \int kdt$$

$$\log N = kt + C$$

$$N = \exp(kt + C)$$

$$N(t) = \exp(kt + C)$$

As we did before, at t=0:

$$N(t=0) = \exp(C) = N_0$$

$$N(t) = N_0 \exp(kt)$$

Cell apoptosis

$$\frac{dN}{dt} = -kN$$

$$N(t) = N_0 \exp(-kt)$$

Ordinary differential equations

$$\frac{dl}{dt} = -k_d$$

$$\frac{dN}{dt} = kN$$

To solve, we need to know one constant ("initial condition")

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Summary

- Biological phenomena can be described mathematically using differential equations
- Polymerization, depolymerization of actin
- Bacterial growth
- Cell apoptosis