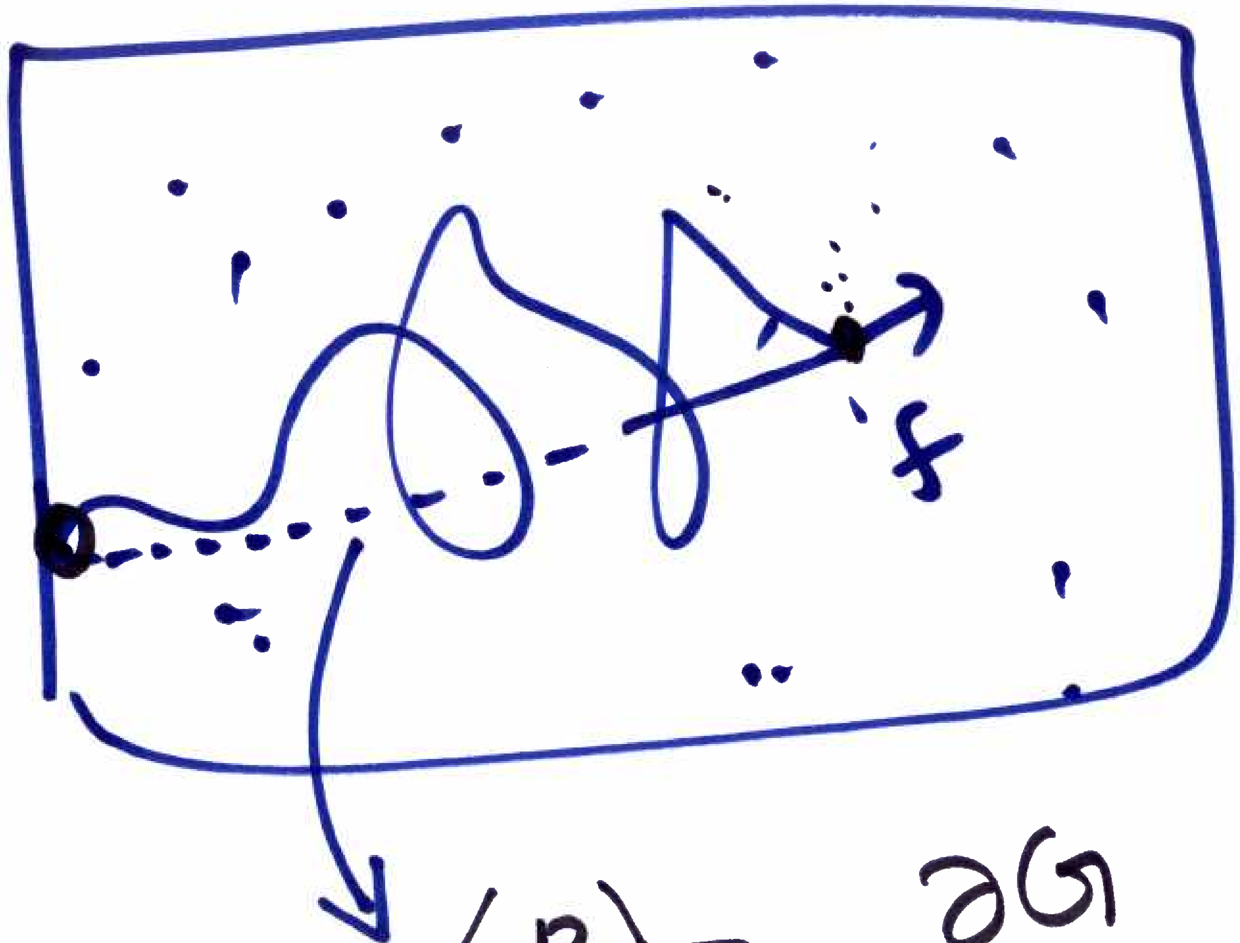
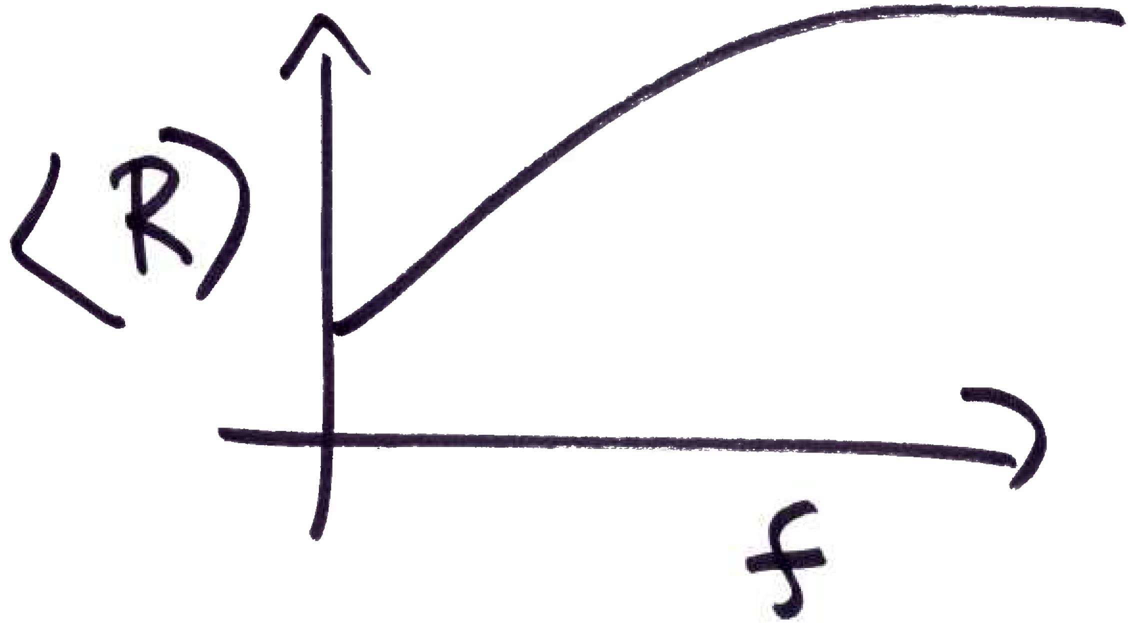
Prof. Ranjith
Date:- 17/05/2012
Lec. 37



$$\langle R \rangle = \frac{\partial G}{\partial f}$$

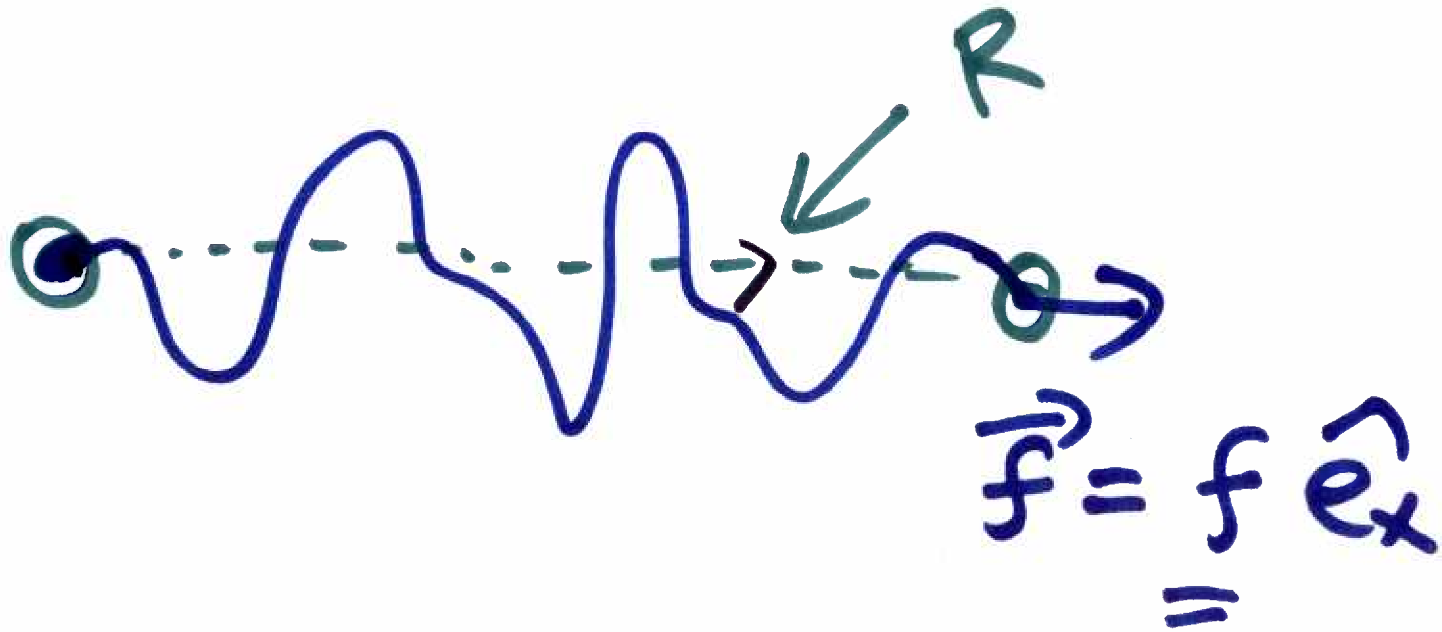
Force-Extension Relation



$$\langle R \rangle = \frac{\partial G}{\partial f}$$

$$G(f) = \,? = -k_B T \ln \underline{\underline{Z}}$$

Free energy $G(f)$ for single-stranded DNA



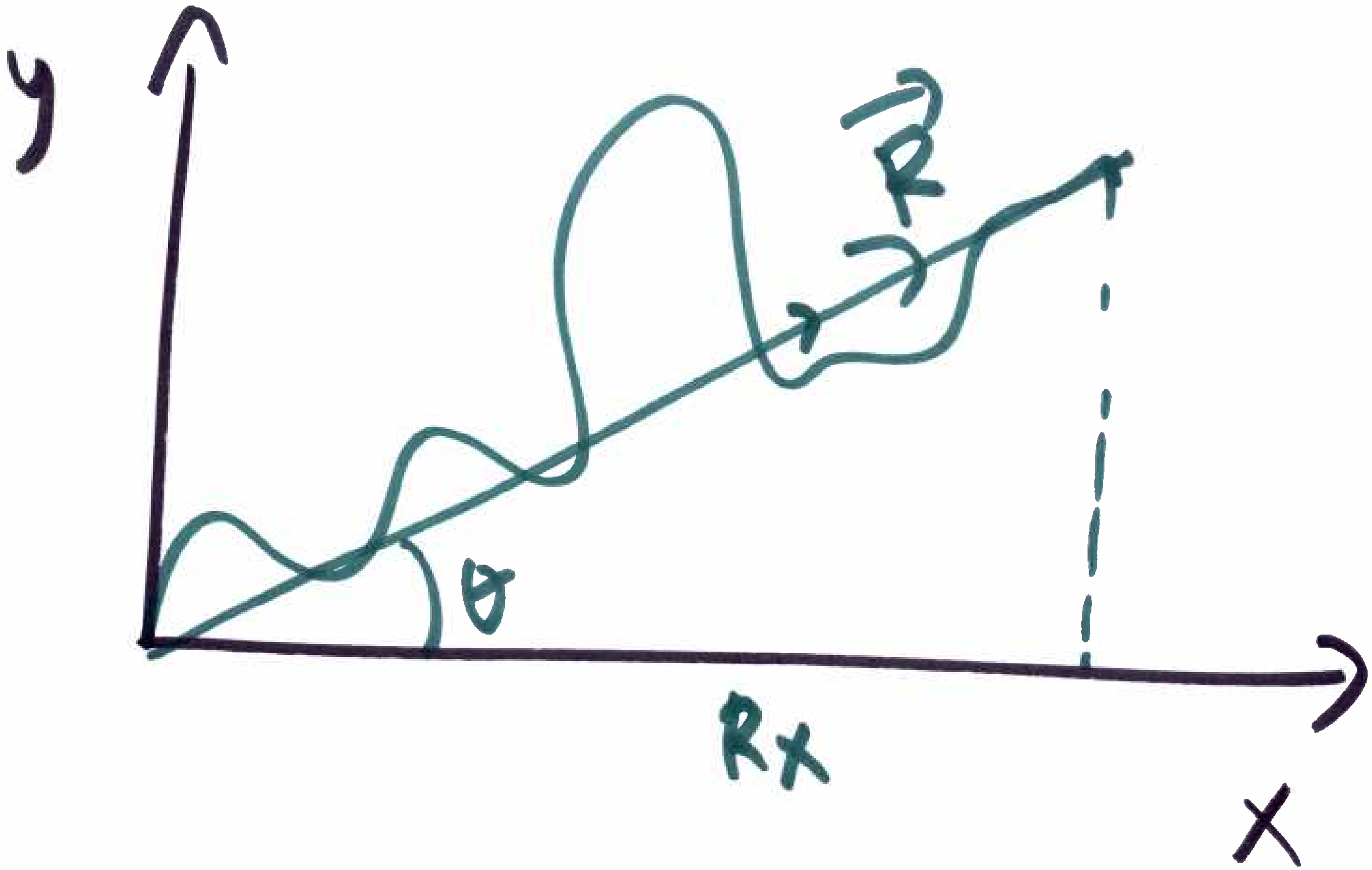
$$E_B = \underline{\underline{A}} \sum_i \left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)$$

$$E_B^{ss} \approx 0$$

$$E = \vec{f} \cdot \vec{R}$$

$$= f\hat{e_x} \cdot \left(R_x \hat{e_x} + R_y \hat{e_y} + R_z \hat{e_z}\right]$$

$$\boxed{E = f R_x}$$



$$E = f R_x = \tilde{f} \cos\theta$$

$$E_B \approx 0$$

$$E_p = \tilde{f} \cos\theta$$

$$E = \tilde{f} \cos\theta$$

$$Z = \sum_i e^{-\beta E_i}$$

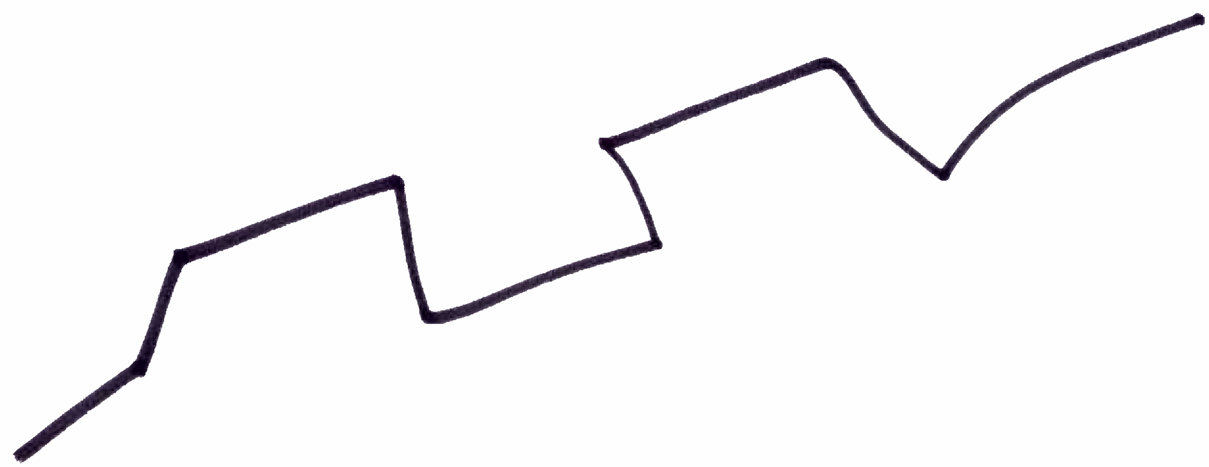
$$= \int \{\text{states}\} \, e^{-\beta E}$$

$$= \int \{dt\} \, e^{-\beta \tilde{f} \cos\theta}$$

$$\int d\vec{t} = \int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\varphi$$

$$\int_{0}^{2\pi} d\varphi \int_{-1}^{1} d\cos\theta \; e^{-\beta \tilde{f} \cos\theta}$$

$$Z = \left[4\pi \frac{\sinh \beta \tilde{f}}{\beta \tilde{f}} \right]^N$$



$$G = -k_B T \ln Z$$

$$= N\left[\ln \frac{\tilde{f}}{k_B T} - \ln \sinh \frac{\tilde{f}}{k_B T}\right]$$

$$G(f) = N\left[\ln \frac{fb}{k_B T} - \ln \sinh \frac{fb}{k_B T}\right]$$

$$\langle R_x \rangle = \frac{\partial G}{\partial f}$$

$$\langle R_x \rangle = Nb\left[\frac{1}{\tanh\frac{fb}{k_BT}} - \frac{k_BT}{fb}\right]$$

$$\frac{fb}{k_BT} \to x$$

flexible polymer

$$\langle R_x \rangle = \frac{1}{\tanh(x)} - \frac{1}{x}$$

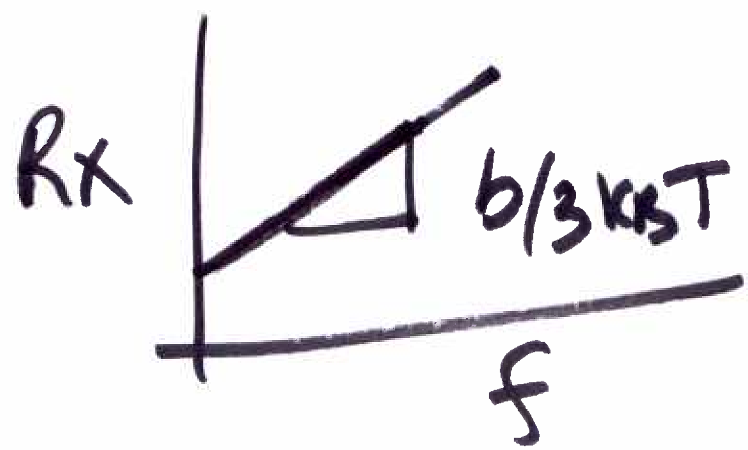
$$x \ll 1 \Rightarrow f \ll \frac{k_B T}{b}$$

$$\langle R_x \rangle \approx \frac{x}{3} + \dots O(x^3)$$

For small force

$$f \ll \frac{k_B T}{b}$$

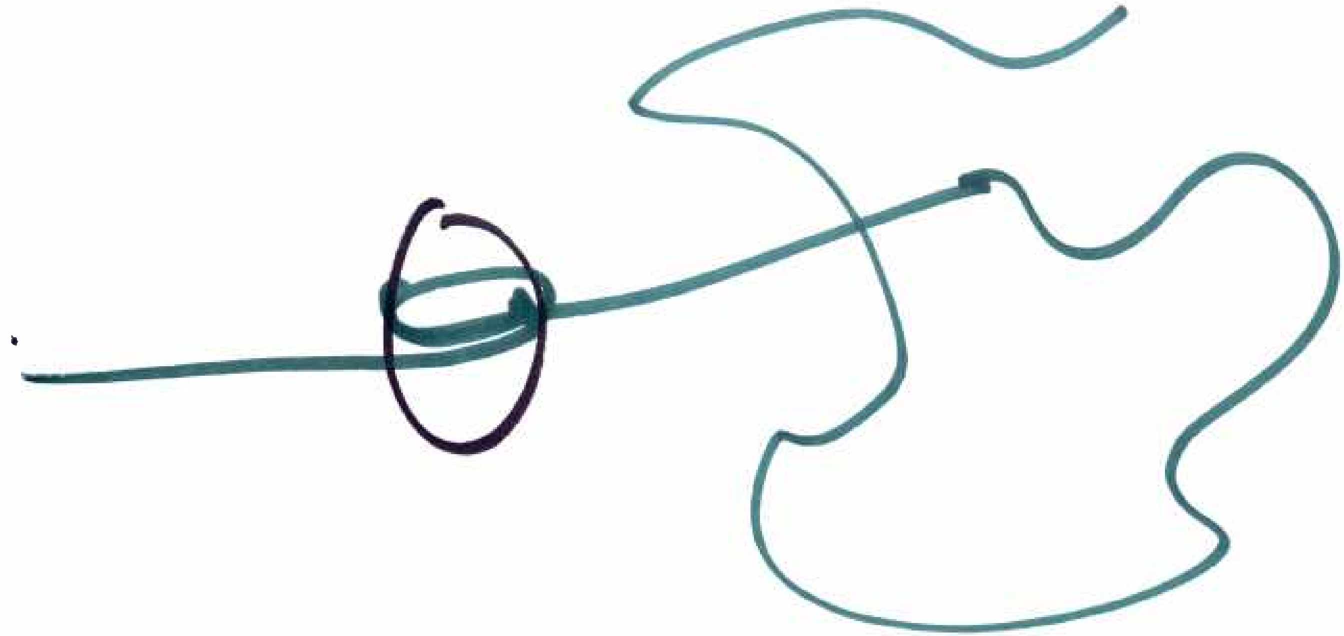
$$\langle R_x \rangle = \frac{x}{3} + \cdots = \frac{fb}{3k_B T}$$



for large f

$$f \gg \frac{k_B T}{b}$$

$$\frac{\langle R_x \rangle}{Nb} = Nb\left[1 - \frac{k_B T}{fb}\right]$$



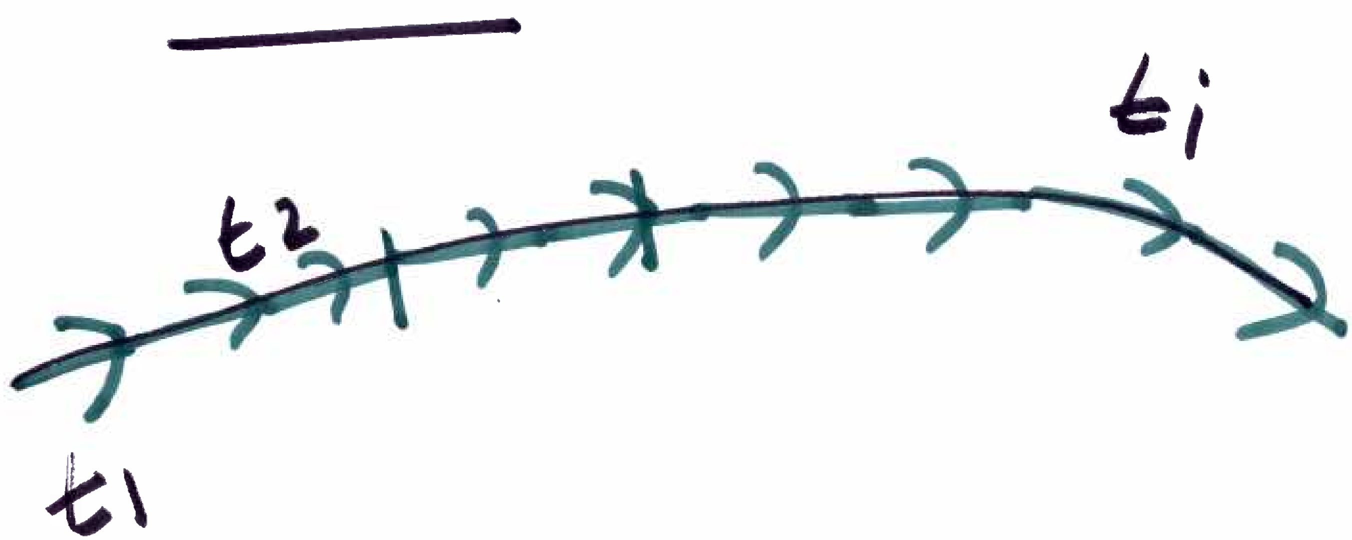

f

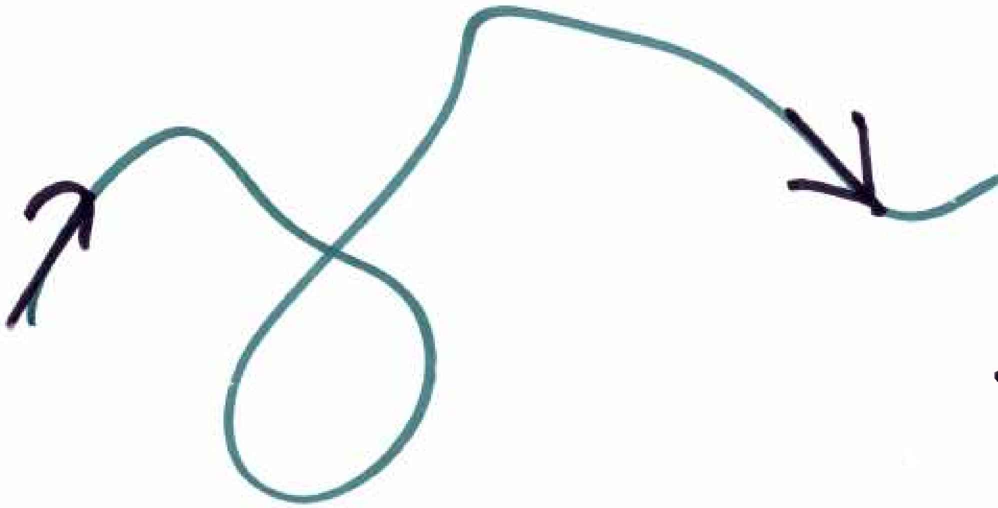
$$E = \int \textcircled{k} \left(\frac{\partial t}{\partial s}\right)^2 ds$$

$$\int \textcircled{k} \left(\frac{\partial^2 r}{\partial s^2}\right)^2 ds$$

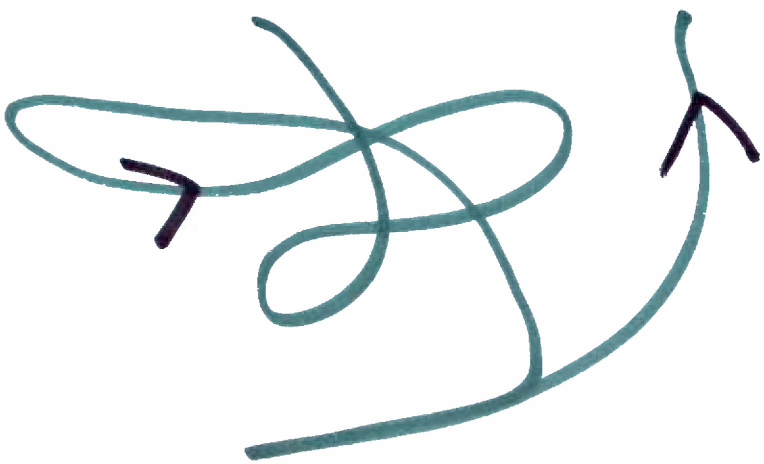
$$E = \textcircled{A} \sum_i \left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)$$

Persistence length





$$\vec{t}_i \cdot \vec{t}_{i+r}$$

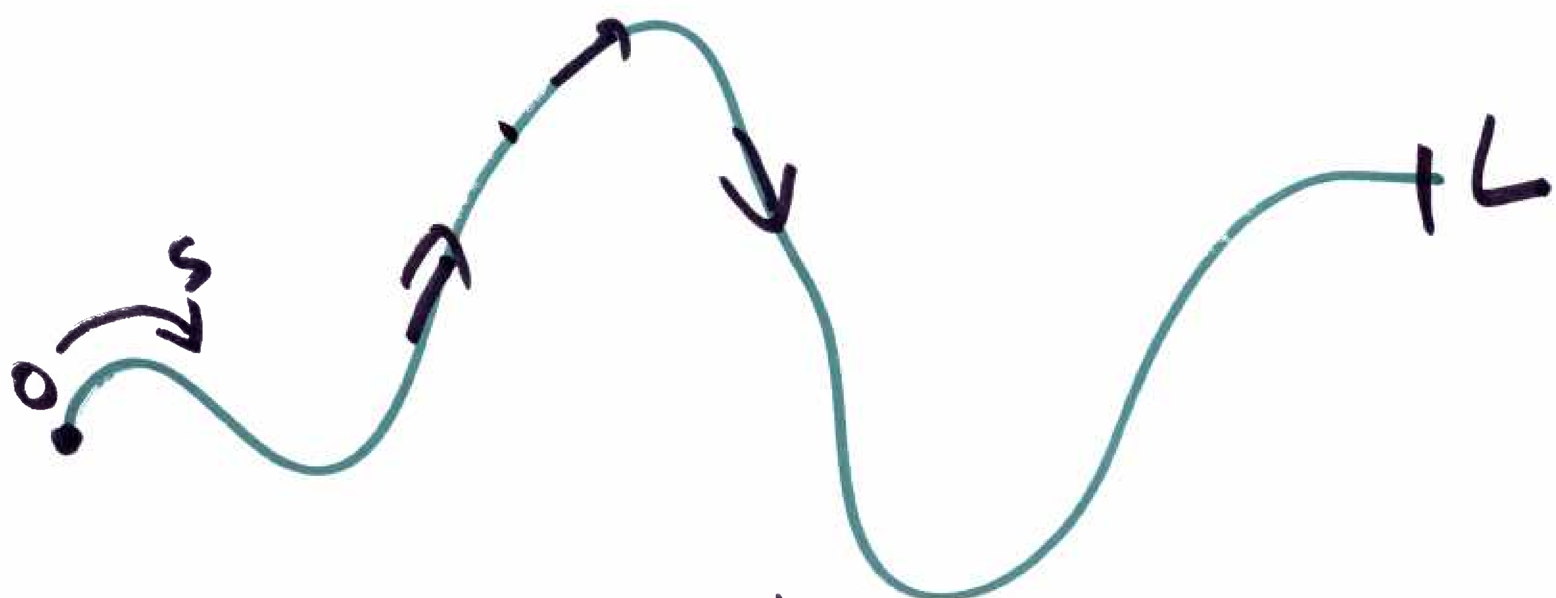


$$\vec{t}_i \cdot \vec{t}_{i+r} = (1) \cos\theta$$

$$= 1$$

$$|t_i| = 1$$

$$\langle \hat{t}(s) \cdot \hat{t}(s+ds) \rangle = e^{-\frac{ds}{L_p}}$$



arc length (s)

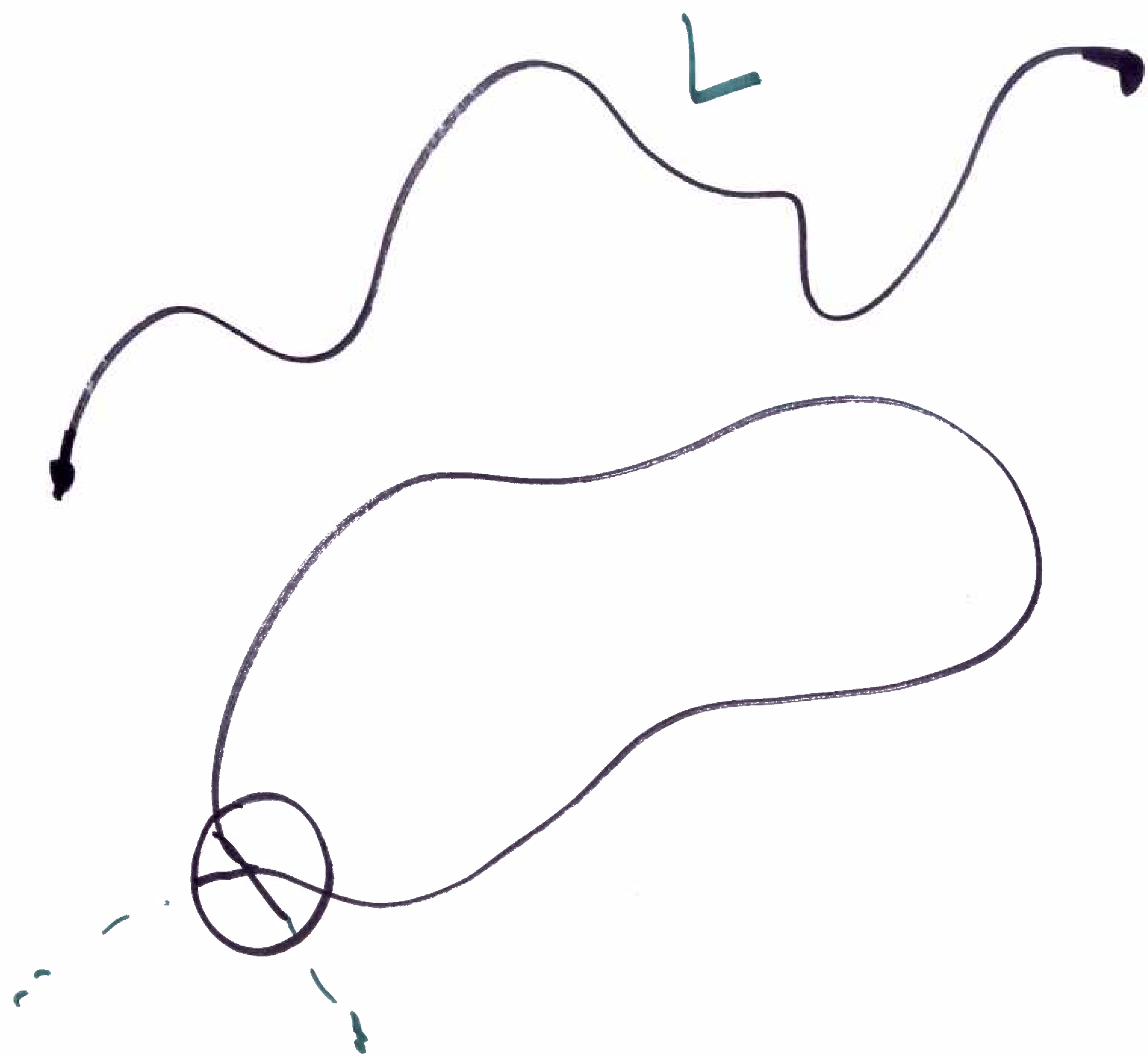
L_p = Persistence length

$$L_p = \frac{K}{k_B T} = \frac{\text{Bending stiffness}}{k_B T}$$

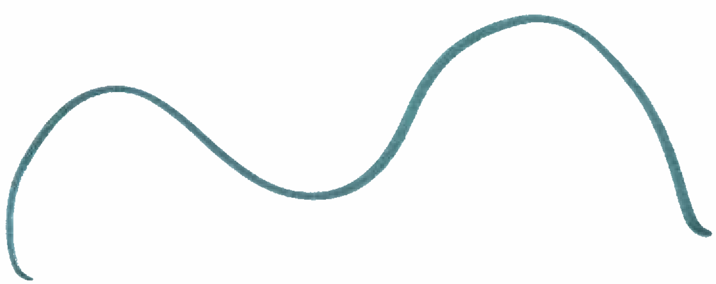
L_p for DNA = 50 nm ≃ 150 bp

L_p fo actin ≈ 10 μm

L_p for Microtubule ≈ mm



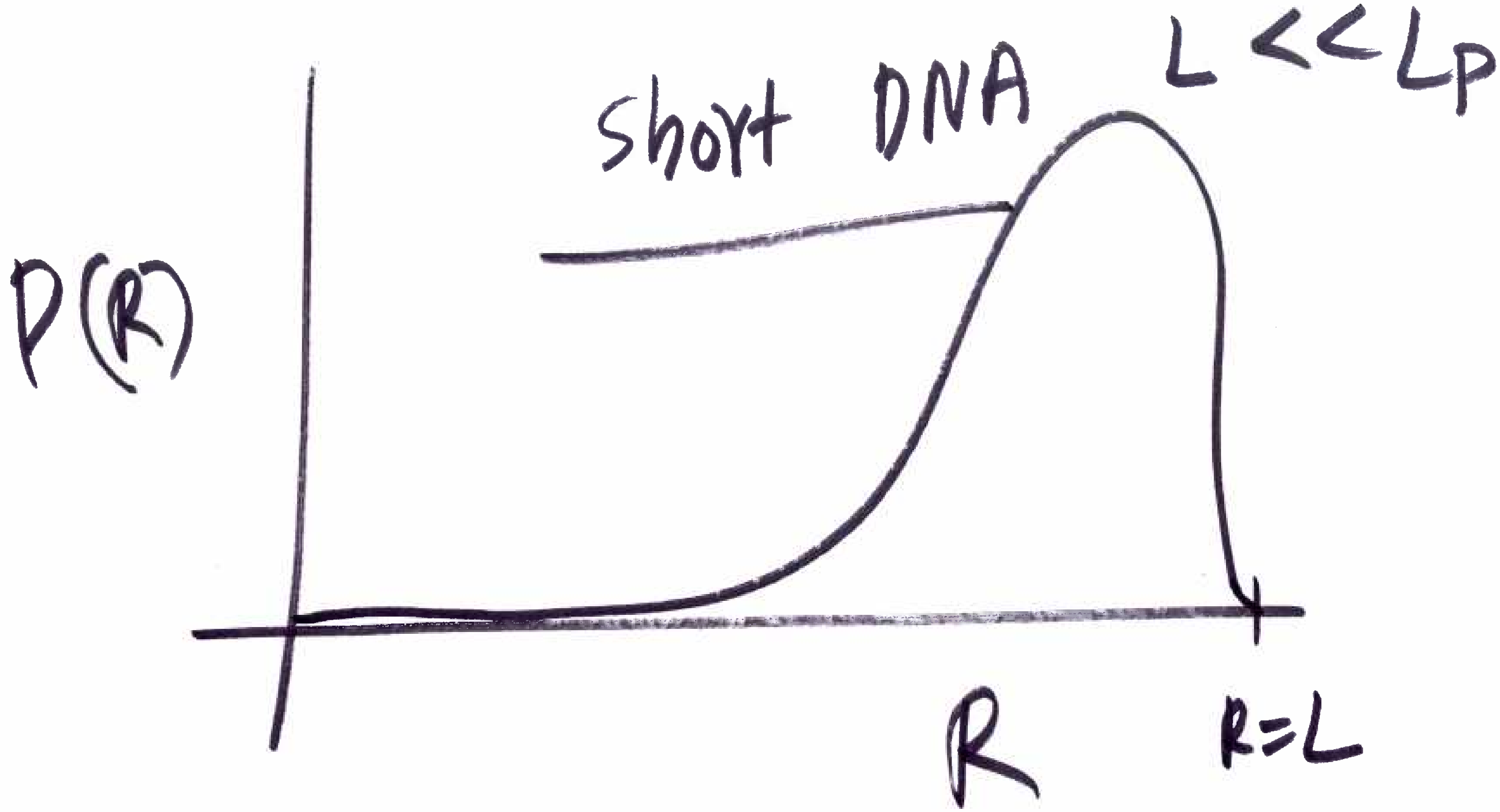
$$Z = \int e^{-\beta E} \{dt\}$$



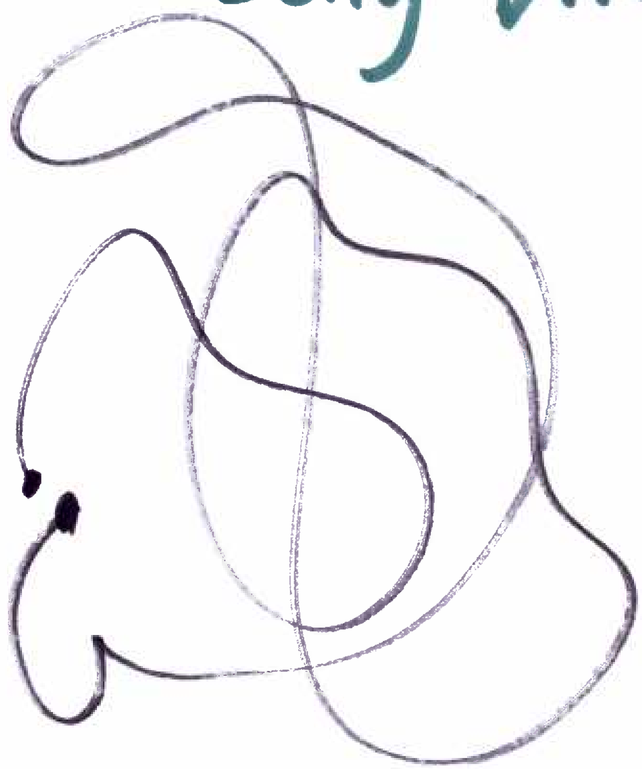
$$P = \frac{e^{-\beta E}}{Z}$$

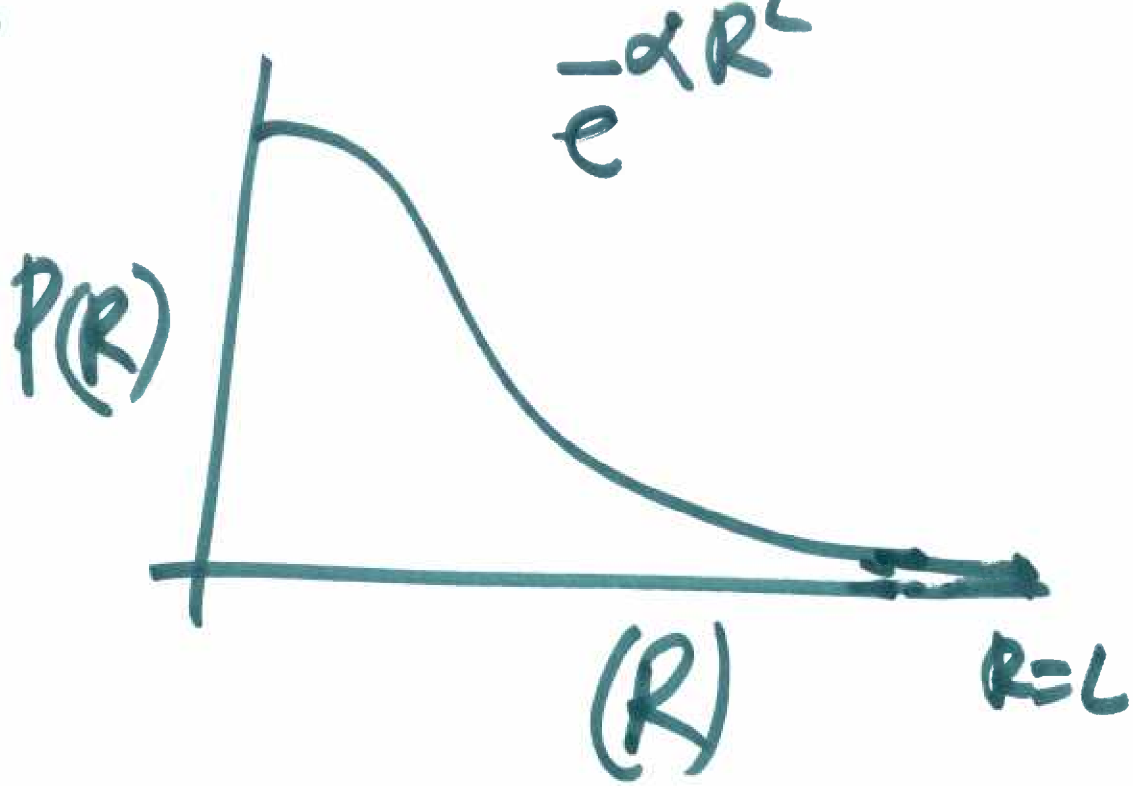
$$= \frac{e^{-\beta E(R=0)}}{Z}$$

$P(R)$ = Probability that DNA will have end-to-end distance = R


Short DNA
L << Lp
P(R)
R
R=L

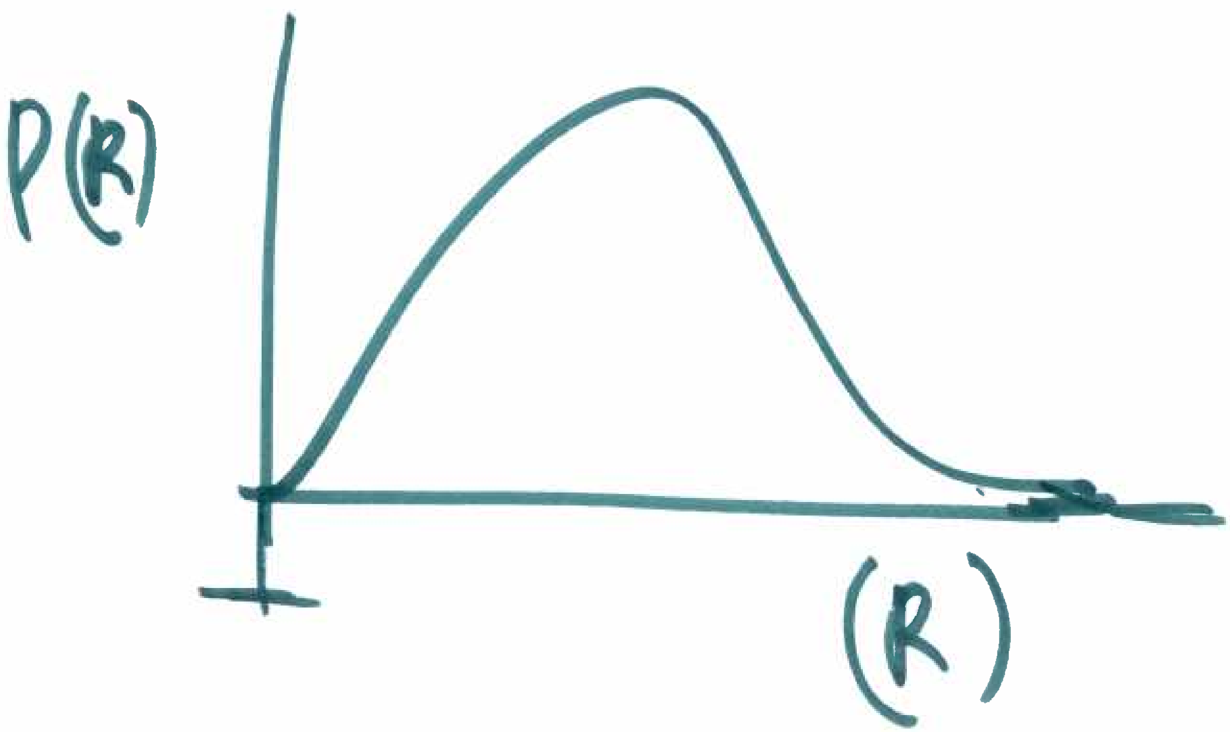
Long DNA

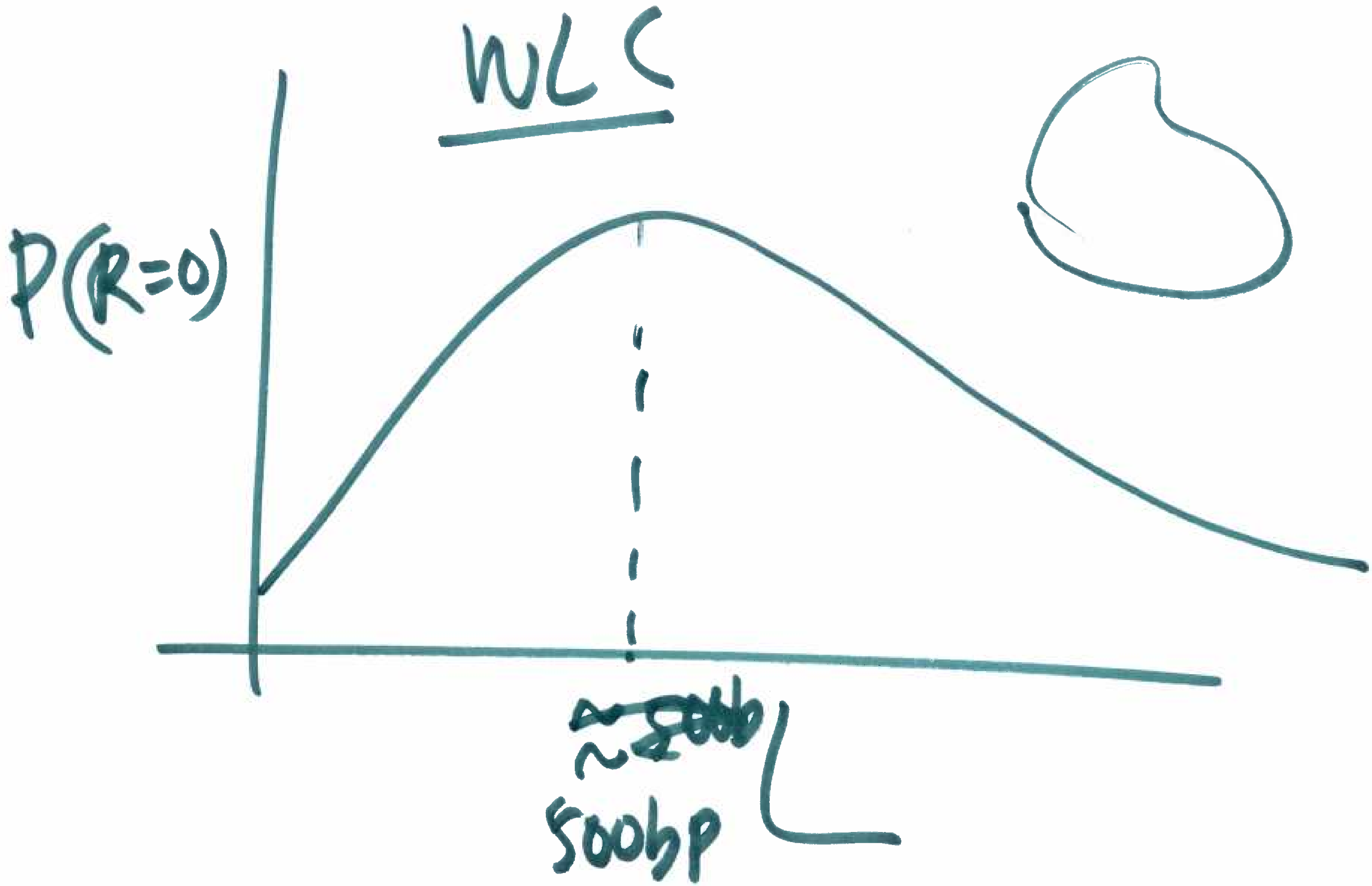




$L >> L_p$

$$L \sim L_p$$




WLC
P(R=0)
~500bp
L

$$G = H - TS$$

$$F = E - TS$$