Prof Ranjith
Lec - 35
(15/5/12)

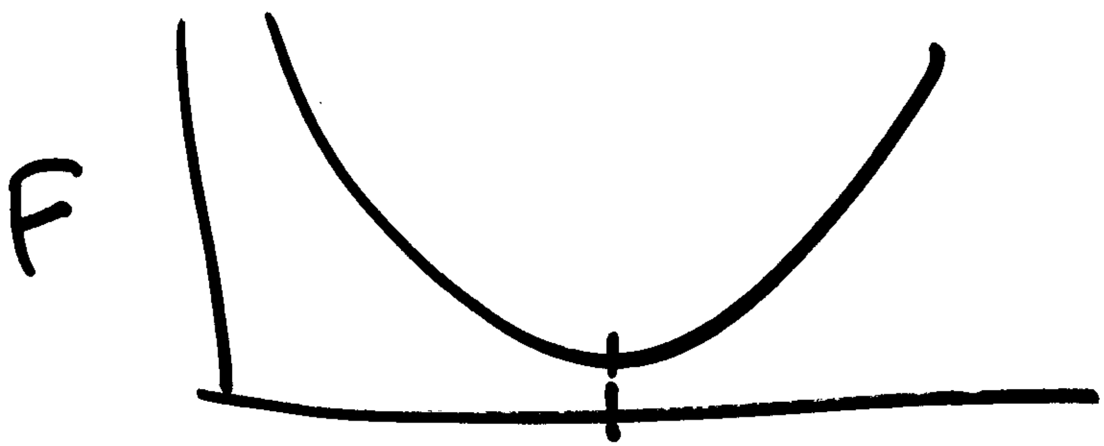
Entropy, $S = +k_B \ln \Omega$

$$S = -k_B \sum_i p_i \ln p_i$$

Free energy

$$F = \breve{\underline{E}} - T\underline{S}$$

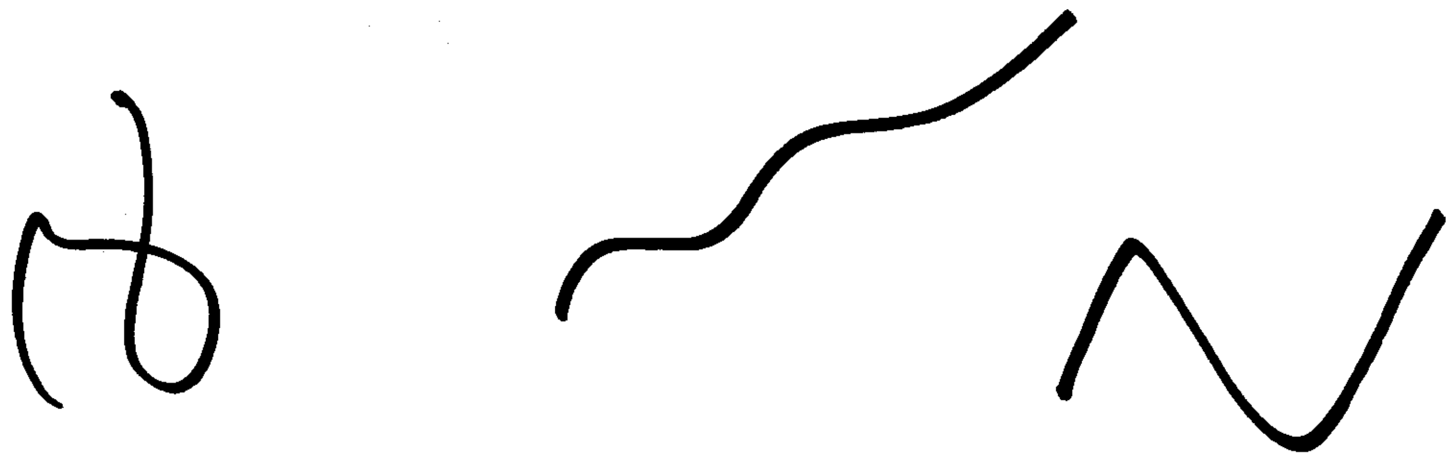
$\Delta F = 0 \Rightarrow$ Equilibrium



Partition function

$$Z = \sum_i e^{-\beta E_i} = \sum_i e^{-\frac{E_i}{k_B T}}$$

$$\beta = \frac{1}{k_B T}$$



E_1 E_2 E_3

$$Z = \sum_{i=1}^{3} e^{-\frac{E_i}{k_B T}} = e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}} + e^{-\frac{E_3}{k_B T}}$$

$$e^{-\beta E_1} + e^{-\beta E_2} + e^{-\beta E_3} = Z$$

$E_1 = -2k_BT$ $E_2 = 0k_BT$ $E_3 = 5k_BT$

$$Z = e^{2} + 1 + e^{-5} = 1 + e^{2} + e^{-5}$$

$$F = -k_B T \ln Z$$

$$F = -k_B T \ln\left(1 + e^{2} + e^{-5}\right)$$

E_1 E_2 E_3

$$F = E - TS$$

$$Z = \sum_i e^{-\beta E_i}$$

If $E_i = 0$ for all i

$Z =$ Total number of 'states'

$$F = -k_B T \ln Z = -\underline{k_B} T \underline{\underline{\ln \Omega}} = -TS$$

$E_1 \qquad E_2 \qquad E_3$

$$Z = e^{-\beta E_1} + e^{-\beta E_2} + e^{-\beta E_3}$$

$$P_i = \frac{e^{-\beta E_i}}{Z}$$

$$p_1 = \frac{e^{-\beta E_1}}{z} \qquad \frac{e^{-\beta E_2}}{z} \qquad \frac{e^{-\beta E_3}}{z}$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$-2k_BT$ $\quad 0 \quad$ $5k_BT$

$$e^{2}+1+e^{-5}$$

$$P_2 = \frac{e^{0}}{e^{2}+1+e^{-5}} = \frac{1}{1+e^{2}+e^{-5}}$$

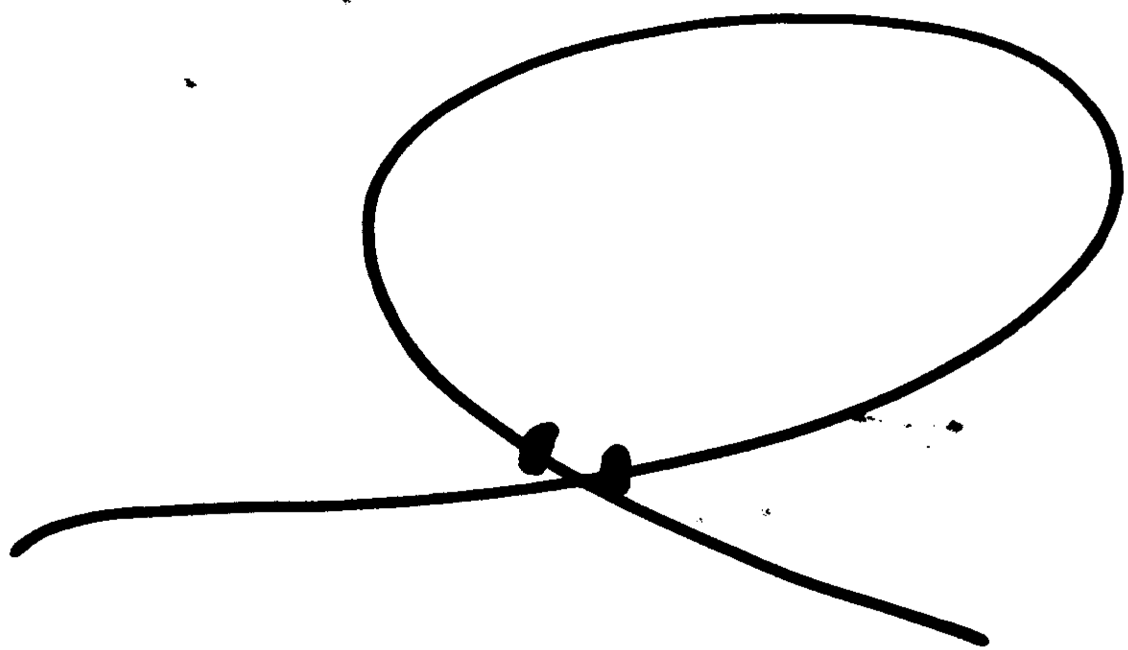
$$P_1 = \frac{e^2}{1 + e^2 + e^{-5}}$$

$$P_2 = \frac{1}{e^2 + e^{-5} + 1}$$

$$P_3 = \frac{e^{-5}}{1 + e^2 + e^{-5}}$$

$$P_1 + P_2 + P_3 = 1$$

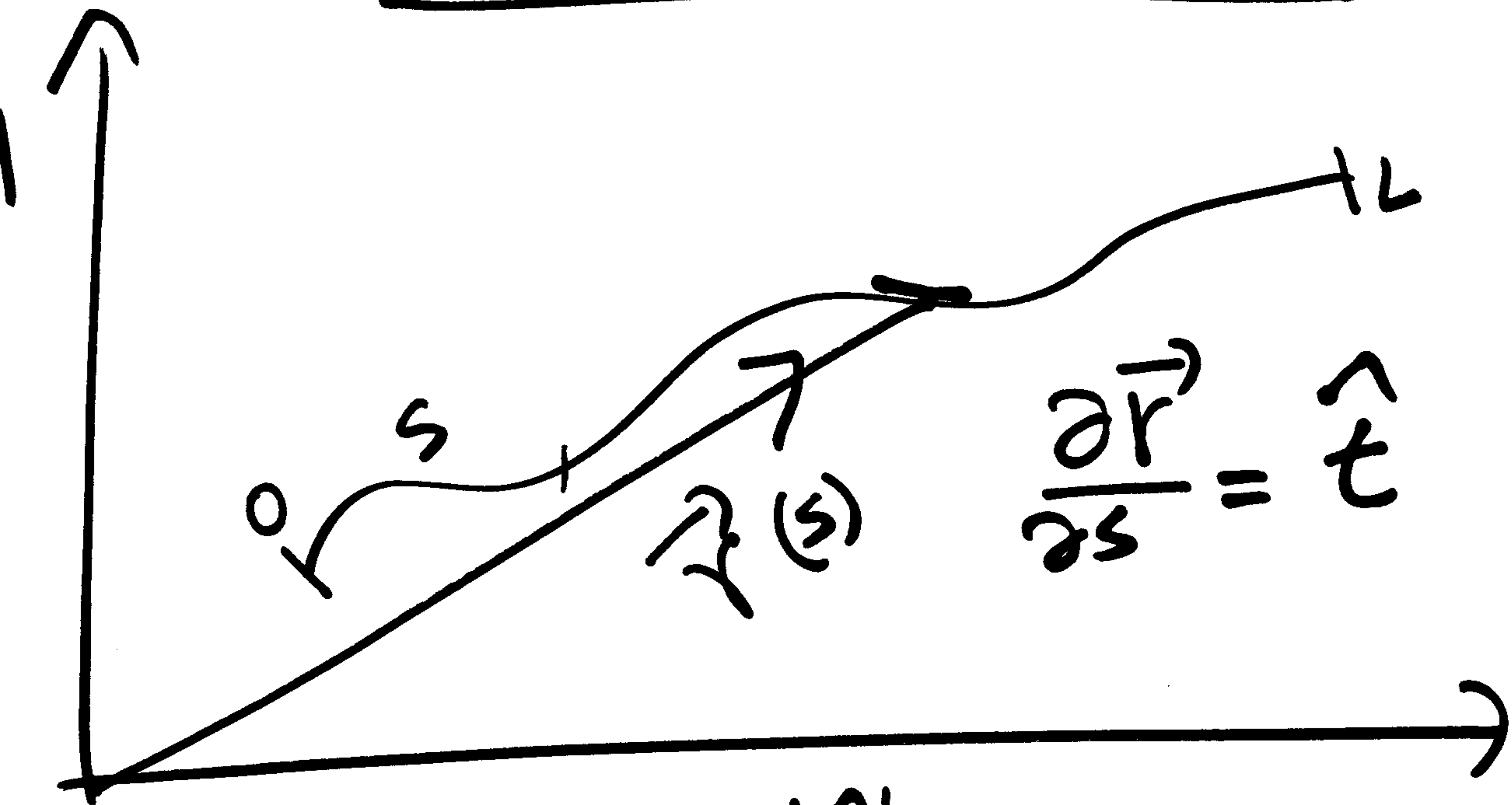
$$S = (P_1 \ln P_1 + P_2 \ln P_2 + P_3 \ln P_3)(-k_B)$$



$$P_c = \frac{e^{-\beta E_c}}{Z}$$

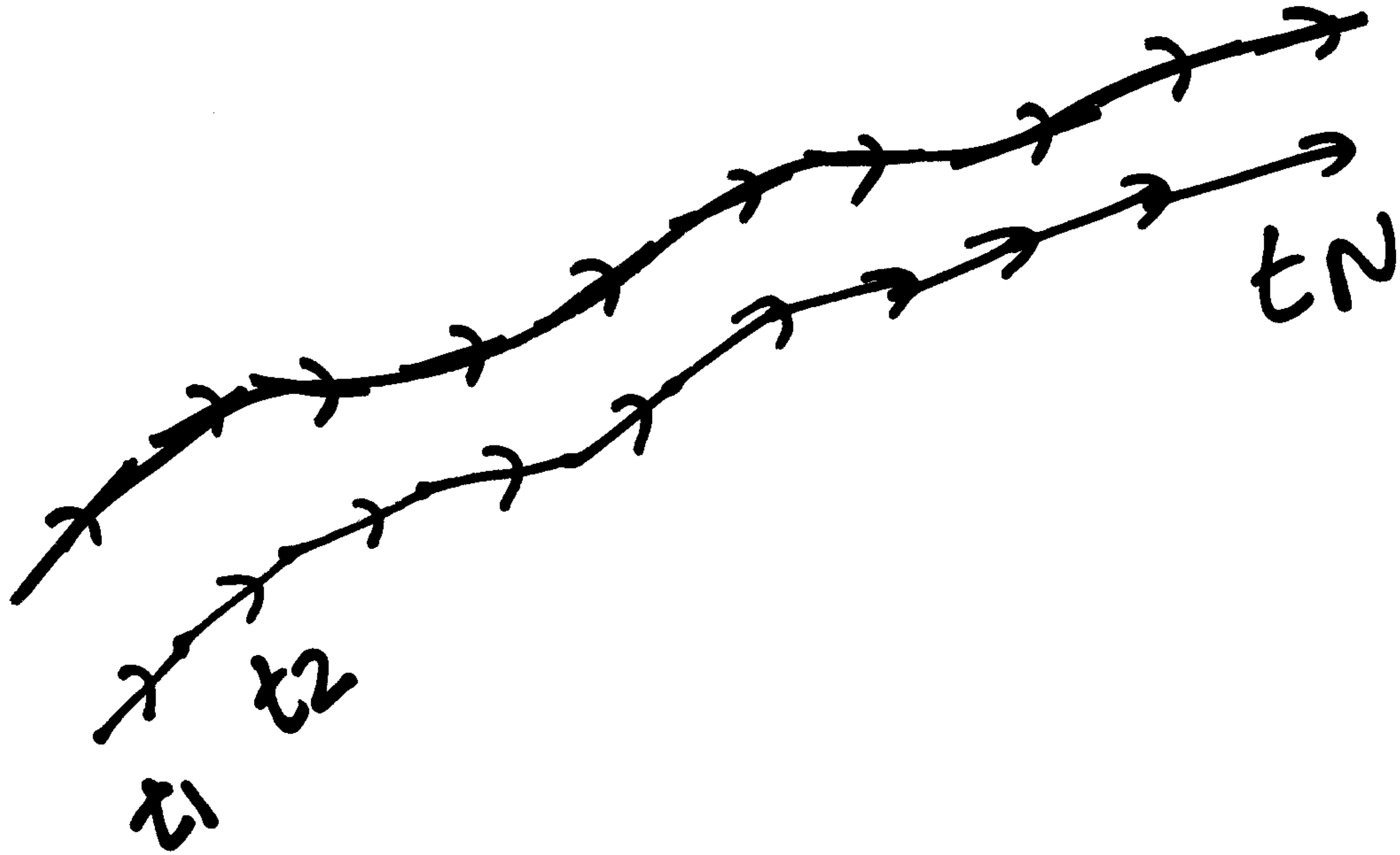
$$\uparrow$$

Worm-like Chain model

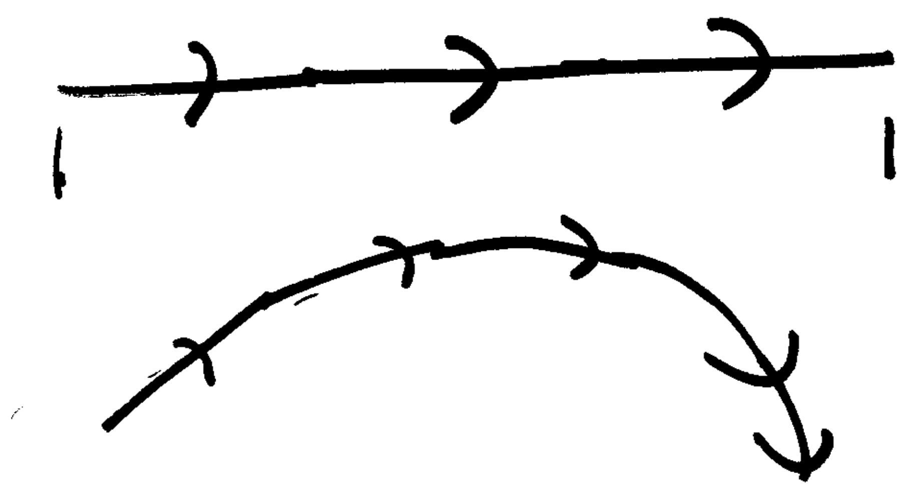


$$\frac{\partial \vec{r}}{\partial s} = \hat{t}$$

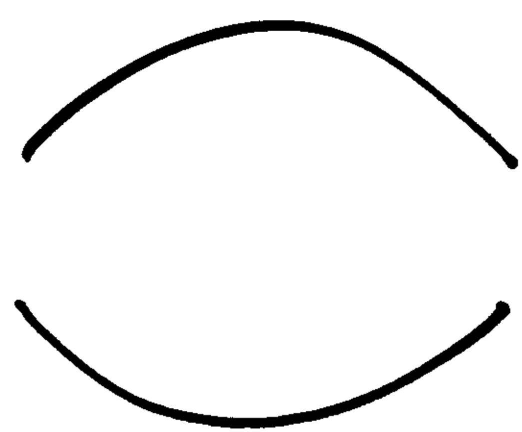
$$|\hat{t}| = 1$$


t1
t2
tN

$$E_B = K \int \underbrace{\left(\frac{\partial \hat{t}}{\partial s}\right)^2}_{\text{curvature}} ds \qquad K = \text{Bending Stiffness}$$



$$\frac{\partial t}{\partial s} = 0$$

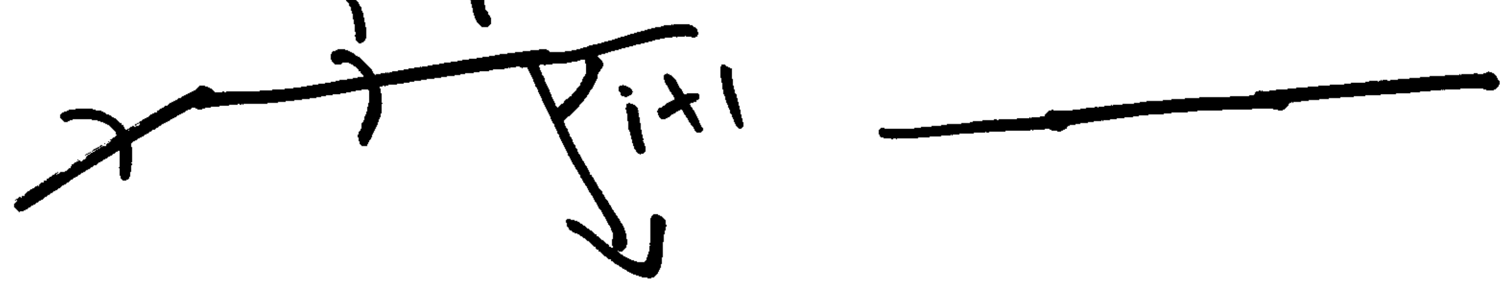


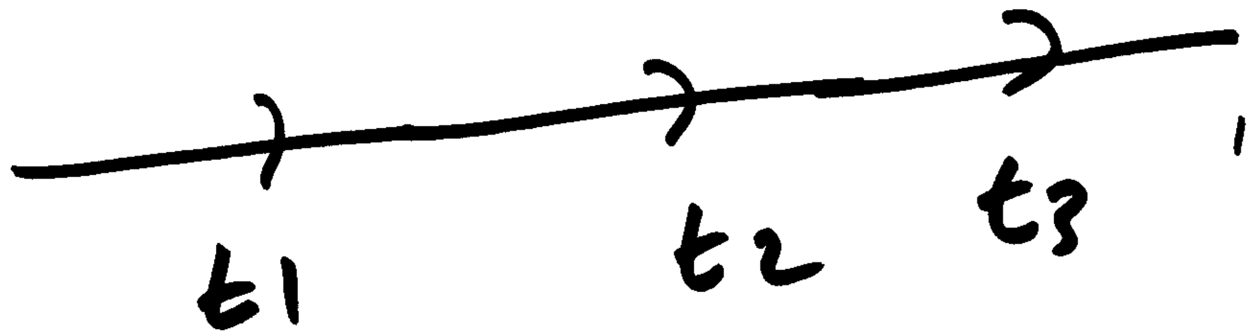
$$\cancel{E = k \sum_i 1 - t_i \cdot t_{i+1}}$$

$$E = A \sum_i \left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)$$

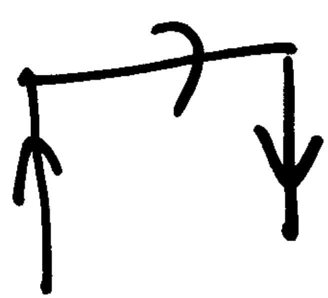
$$A \sum_i \left(1 - \cos\theta_i\right)$$

$$A_i \sum_i$$

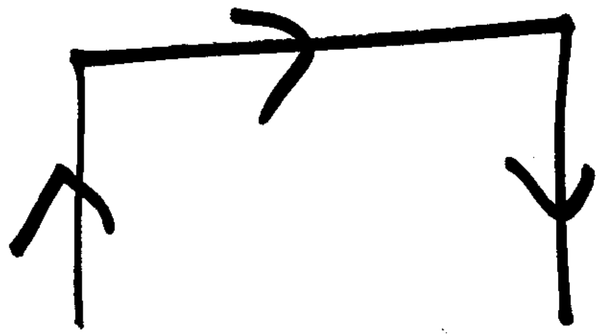




$$E = A\sum_{i} \left(1 - \hat{t}_i \, \hat{t}_{i+1}\right) = 0$$



$$E = A\left[(1 - \cos 90) + (1 - \cos 90) \right]$$



$$E = +\left[A\left(1-\cos\frac{\pi}{2}\right) + A\left(1-\cos\frac{\pi}{2}\right)\right]$$

$$= +A + +A = +2A$$

$$E_B = K \int_0^L ds \left(\frac{\partial t}{\partial s}\right)^2$$

$$E_B = A \sum_{i=1}^{N} \left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)$$

$$Z = \sum_i e^{-\beta E_i}$$

$$= \int \{dt\}\, e^{-\beta E(\{t\})}$$

$\{t_i\}$

$$\int d\vec{t}_1 \int d\vec{t}_2 \int d\vec{t}_3 \cdots \int d\vec{t}_N \, e^{-\beta E}$$

$$E = A \sum_i \left(1 - \vec{t}_i \cdot \vec{t}_{i+1}\right)$$

$\int d\Omega$

$$\int d\Omega_1 = \int_0^{2\pi} d\varphi \int_0^{\pi} d\cos\theta$$