Biomathematics Assigments

Modules I

1. Imagine a farmer doing fish farming. He first introduces a small number of fish into a pond and supplies sufficient food. After a few days the pond is full of fish. Plot a typical growth curve: i. e., plot number density (number per unit volume), $y(t)$ as a function of time. Below this curve, plot the corresponding derivative: $\frac{dy}{dt}$. When is maximum yield ?

2. Find derivatives ($\frac{df}{dx}$) of the following functions

   $f(x) = x^{23}, \qquad f(x) = x^{-8}, \qquad f(x) = \sqrt{x}$

   $f(x) = x^{1/8},\ f(x) = \frac{1}{\sqrt{x}},\ f(x) = x \ln x + (1-x)\ln(1-x)$

   $f(x) = \sin 2x,\ f(x) = \cos^2 x,\ f(x) = \exp(kx)$

   $f(x) = x\exp(-2x),\ f(x) = \frac{8x-4}{x^2 \sin x}\ f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

3. Calculate the following integrals (If the limits are not given, they are indefinite integrals):

   $\int x^9 dx, \quad \int x^{-2} dx, \quad \int \frac{1}{x} dx,\ \int_0^{2\pi} \sin x dx$

   $\int x^{1/2} dx, \quad \int \cos(3x) dx, \quad \int_0^{\pi} \sin x dx$

   $\int_{-1}^{1} \exp(-kx) dx, \quad \int_{-\infty}^{\infty} x\exp(-kx) dx$

4. Consider the function $y(x) = x$

   (a) Plot function in the range [-5,5] (i. e. values of x between -5 and 5)

   (b) Looking at the plot, without doing integration, write down the answer for

   $$\int_{-5}^{+5} x dx$$

You need to explain how you obtained the answer.

(c) Do the integration $\int_{-5}^{+5} x dx$ and find the answer.

5. Consider the function

$$f(x) = x \exp(-kx),$$

where k is a positive number.

(a) Find out values at $f(x \to 0)$ and $f(x \to \infty)$

(b) Find out if f(x) has any maximum or minimum

(c) If yes, find out the value of x where there is a maximum or minimum

(d) find out whether it is a maximum or minimum

(e) Schematically plot the function in the range [0,∞] (i. e. values of x should be between 0 and ∞)

6. Plot the following functions

(a) $f(x) = x^2 \exp(-kx^2)$

(b) $f(x) = \frac{\sin x}{x}$

Modules II & III

1. Consider proteins binding onto DNA. Let $\rho(t)$ be the mean density of proteins at any given time t.

(a) If one considers only dissociation of proteins (no binding), how would the mean density change ? Write down an equation for mean density change ($\frac{d\rho}{dt}$) in this case, assuming the dissociation rate as k_{off}.

(b) Write down an equation for the mean density change ($\frac{d\rho}{dt}$), if one takes only binding; consider binding rate k_{on} and assume that there is no dissociation. Hint: Once the DNA is fully covered with proteins, then no more protein can bind.

(c) Write down an equation for the mean density change ($\frac{d\rho}{dt}$) when there are both binding and dissociation

(d) After sometime, the system will reach an "equilibrium" where the mean density is a constant. At equilibrium what will be the mean density ?

2. Imagine a protein which is like 3 balls arranged on a line, and connected by springs. Let x_1 be the position of the first ball, x_2 be the position of the second ball and x_3 and be the position of the third ball. Energy of the system is given by

$$E = \frac{k}{2}(x_2 - x_1)^2 + \frac{k}{2}(x_3 - x_2)^2$$

Let the force on the *second ball* be given by

$$\vec{f} = -\frac{\partial E}{\partial x_2}\hat{x}$$

(a) Calculate $\vec{f}$

(b) Write down values for x_1, x_2 and x_3 such that $f = 0$.

(c) Write down values for x_1, x_2 and x_3 such that the second ball is pulled towards the positive x-direction

3. Imagine 2 molecules interacting via the Lennard-Jones potential; the energy of the system is given by

$$V = \epsilon\left(\frac{2^{12}}{(\vec{r}_1 - \vec{r}_2)^{12}} - \frac{2^7}{(\vec{r}_1 - \vec{r}_2)^6}\right)$$

where $\vec{r}_1$ and $\vec{r}_2$ are position vectors of the molecules and ϵ is a constant. Calculate the force on the first molecule

$$\vec{f}_1 = -\frac{\partial V}{\partial \vec{r}_1}$$

Also Calculate the force on the second molecule

$$\vec{f}_2 = -\frac{\partial V}{\partial \vec{r}_2}$$

4. If there are three molecules, the total energy will be

$$V = \epsilon\left(\frac{2^{12}}{(\vec{r}_1-\vec{r}_2)^{12}} - \frac{2^7}{(\vec{r}_1-\vec{r}_2)^6}\right) + \epsilon\left(\frac{2^{12}}{(\vec{r}_1-\vec{r}_3)^{12}} - \frac{2^7}{(\vec{r}_1-\vec{r}_3)^6}\right) + \epsilon\left(\frac{2^{12}}{(\vec{r}_2-\vec{r}_3)^{12}} - \frac{2^7}{(\vec{r}_2-\vec{r}_3)^6}\right)$$

Calculate

$\vec{f}_1 = -\frac{\partial V}{\partial \vec{r}_1}$, $\vec{f}_2 = -\frac{\partial V}{\partial \vec{r}_2}$, $\vec{f}_3 = -\frac{\partial V}{\partial \vec{r}_3}$

5. In the same line as above, imagine 3 charges interacting via Coulomb potential; write down the total electrostatic potential energy in terms position vectors $\vec{r}_1$ and $\vec{r}_2$ and $\vec{r}_3$. Calculate $\vec{f}_1$, $\vec{f}_2$ and $\vec{f}_3$. This is a generalisation of a specific problem done in one of the lectures.

Modules IV , V & VI

1. (a) Globular-actin (G-actin) is a protein with approximately 3 nm radius. What is the diffusion coefficient of actin in water, at 300 K ? (viscosity of water = 0.001 Pascal Second).

   (b) How far will an actin monomer diffuse in 1 minute ?

2. Read about "Central limit theorem".

3. Imagine a person walking N steps, each of size b, in random directions. Let $\vec{R} = R_x\hat{x} + R_x\hat{y}$ be the vector from starting point of the walk to the end point. Write a simple program for calculating end-to-end vector distribution. Calculate $P(|\vec{R}|)$ and $P(R_x)$. Are they different ?

4. As you know, during dynamic instability, microtubule length fluctuates wildly. The fluctuation in length is so large that the standard deviation is equal to the average length of the microtubule.

   (a) What will be the distribution (i. e., what will be the mathematical function one can use for the length distribution of microtubles)?

(b) Write down the exact expression for the length distribution such that the average length and standard deviation are 2μm.

5. Consider a molecular motor moving along a microtubule; assume that the position of the motor (x) changes with time t linearly. That is,

$$x(t) = vt + x_0,$$

where v is the speed of the molecular motor and x_0 is the initial position. Imagine that you did an experiment to measure the position of the motor as a function of time, and you got the following data

Time (s)	Position (μm)
10	30
20	50
30	100

Now, you are asked to fit a straight line to this data. How will you proceed ?

6. Calculate the following Fourier transform:

$$g(k) = \int_{-\infty}^{+\infty} dx \exp\left(-kx^2\right) \exp\left(ikx\right)$$

Modules VII, VIII & IX

1. Write down master equations representing a biased random walker (same as master equation for polymerisation dynamics in Lecture 30). That is, equation for $P(n)$, probability that you will find a walker/polymer/motor at location n. If you assume that n is a continuous variable, you can do taylor expansion and find out $P(n + \Delta n)$. Identifying finite differences as derivatives (see the hint given below), show that the master equation will take the form of a diffusion equation with a bias. Hint

$$\frac{dP}{dn} = \frac{P(n + \Delta n) - P(n)}{\Delta n}$$

(Also figure how to write similar finite difference formula for second derivative)

2. Imagine a population of N haploid individuals. Let us also imagine that there are only two kinds of genes(gene a or gene b). Each individual either carries gene a or gene b. Let p be the probability of of finding gene a in the current generation. According to the Wright-Fisher model:

   (a) What is the probability of finding m number of a genes in the next generation ?

   (b) If N=4, and $p = 0.1$, what is the probability that the gene a will be completely lost in the next generation (probability of not finding the gene a)

3. Read the following paper:
   Quantitative model for gene regulation by λ phage repressor, Ackers, Johnson and Shea, **Proc. Natl. Acad. Sci. USA, Vol 79, Pages: 1129–1133, (1982)** . Note the way the λ operator is written as a system with 8 "states". Write down the partition function for the system. Compute probabilities of finding the system in all the 8 states.

4. Read the following paper:
   Life at low Reynolds number, E. M. Purcell, **American Journal of Physics, Vol. 45, Pages: 3–11, (1977)**