Lecture – 11

Overview of Flight Dynamics – I

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Point Mass Dynamics

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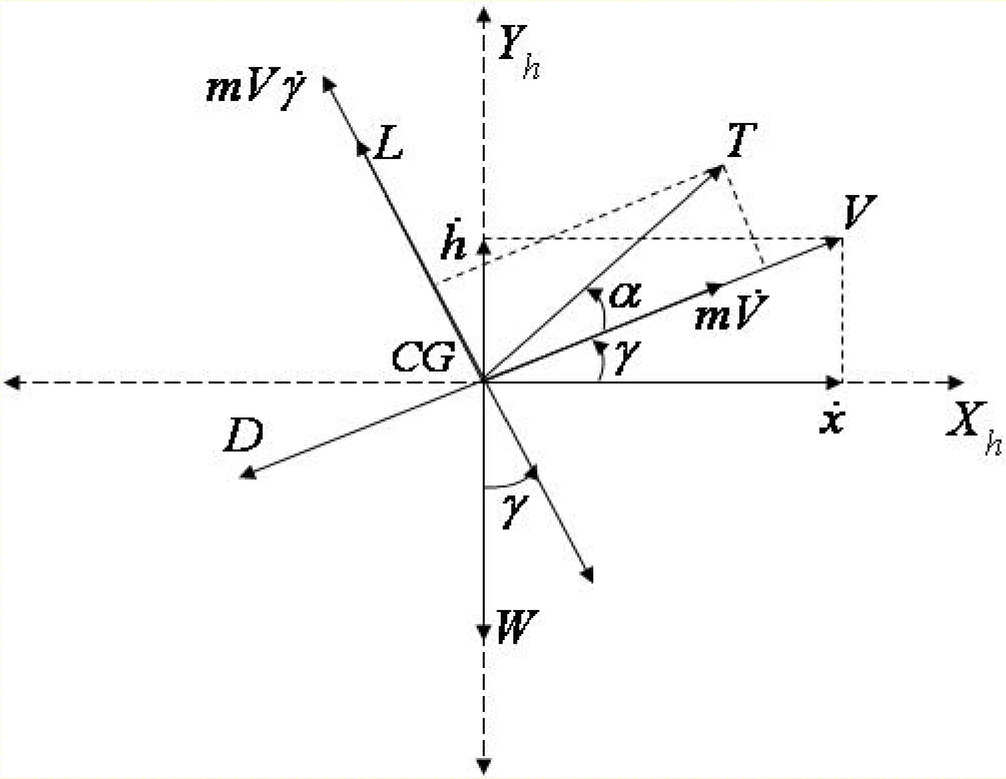
Basic Assumptions for Point Mass Model

1. All forces act at the CG of the airplane
2. Acceleration due to gravity (g) is constant
3. Atmosphere is at rest relative to earth
4. Atmospheric properties are functions of altitude only
5. Forces acting on airplane are thrust, aerodynamic forces and its weight
6. Vehicle attitude is ignored…only direction of velocity vector is considered.

Point Mass Model for Flat (and Non-rotating) Earth



Kinematic Equations

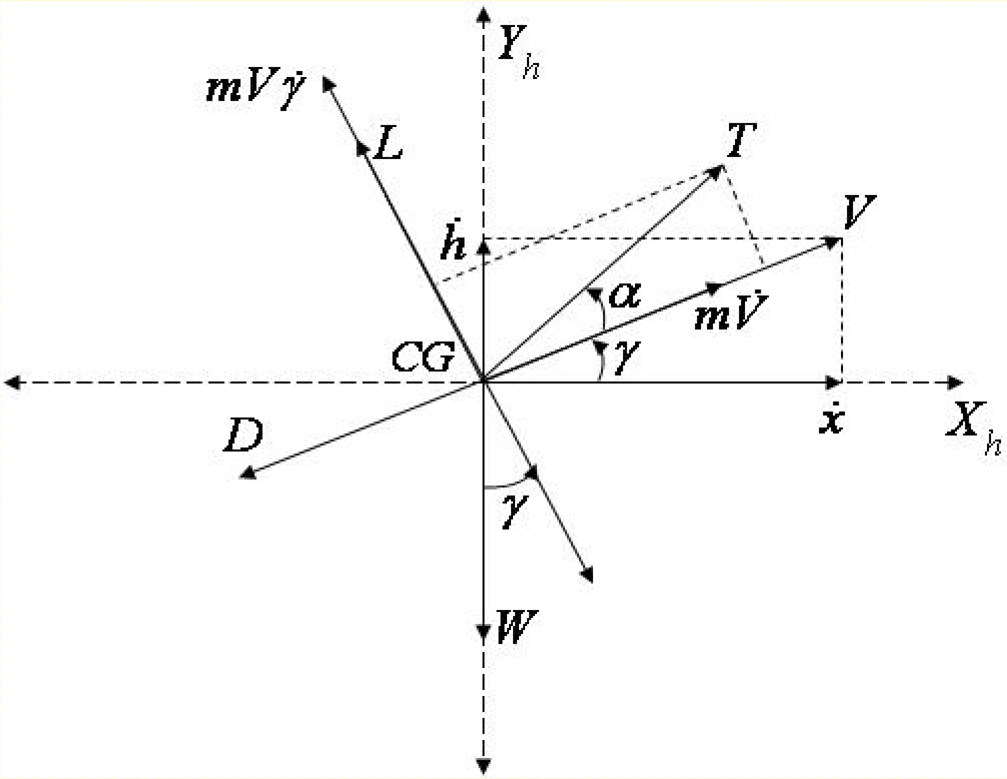
Resolving the velocity vector along the local horizontal

$$\dot{x} = V\cos\gamma$$

Resolving the velocity vector along the local vertical

$$\dot{h} = V\sin\gamma$$

Point Mass Model for Flat (and Non-rotating) Earth



Dynamic Equations

Resolving forces along the velocity vector

$$m\dot{V} = T\cos\alpha - D - W\sin\gamma$$

Resolving forces $\perp$ to the velocity vector

$$mV\dot{\gamma} = T\sin\alpha + L - W\cos\gamma$$

Point Mass Model for Flat (and Non-rotating) Earth

$$\dot{x} = V \cos\gamma$$

$$\dot{h} = V \sin\gamma$$

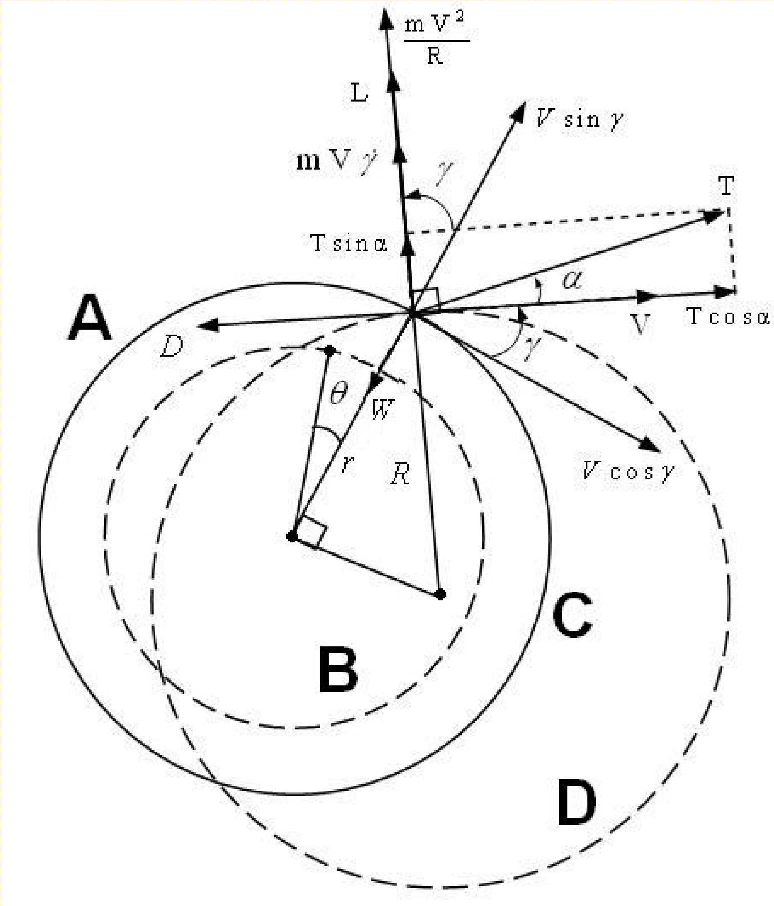
$$\dot{V} = \frac{1}{m}\left(T\cos\alpha - D - mg\sin\gamma\right)$$

$$\dot{\gamma} = \frac{1}{mV}\left(T\sin\alpha + L - mg\cos\gamma\right)$$

Note :

$\dot{x}$ equation is not coupled with others.

Point Mass Model for Spherical, Non-Rotating Earth



Kinematic Equations

Using the force balance equation

$$\frac{m(V\cos\gamma)^2}{r} = \frac{mV^2}{R}\cos\gamma$$

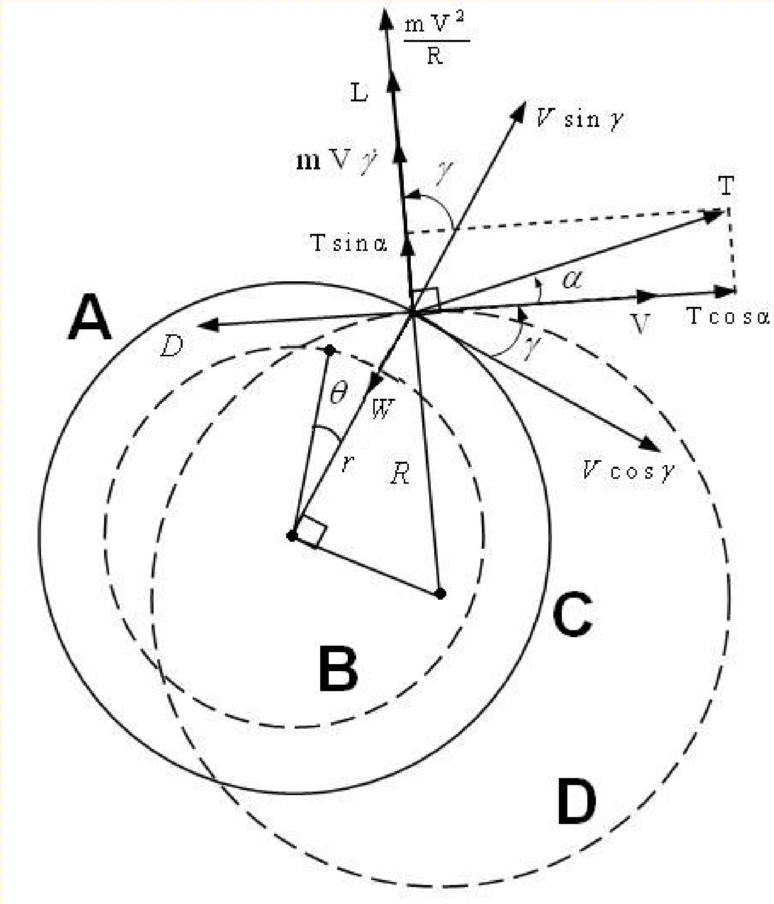
$$R = \frac{r}{\cos\gamma}$$

Resolving velocity vector along local vertical and horizontal

$$\dot{r} = V\sin\gamma$$

$$r\dot{\theta} = V\cos\gamma$$

Point Mass Model for Spherical, Non-Rotating Earth



Dynamic Equations

Resolving force components along the velocity vector

$$m\dot{V} = T\cos\alpha - D - mg\sin\gamma$$

Resolving the force components ⊥ to the velocity vector

$$mV\dot{\gamma} = T\sin\alpha + L - mg\cos\gamma + \frac{mV^2}{R}$$

Point Mass Model for Spherical, Non-Rotating Earth

$$\dot{r} = V \sin\gamma$$

$$\dot{\theta} = \frac{V\cos\gamma}{r}$$

$$\dot{V} = \frac{1}{m}\left(T\cos\alpha - D - mg\sin\gamma\right)$$

$$\dot{\gamma} = \frac{1}{mV}\left(T\sin\alpha + L - mg\cos\gamma + \frac{mV^2}{R}\right)$$

Note:

$\dot{\theta}$ equation is not coupled with others.

Point Mass Model for Spherical and Rotating Earth

When thrust is absent, the kinematic and dynamic equations are

$$\dot{r} = V\sin\gamma \quad , \quad \dot{\varphi} = \frac{V\cos\gamma\sin\psi}{r} \quad , \quad \dot{\theta} = \frac{V\cos\gamma\cos\psi}{r\cos\varphi}$$

$$\dot{V} = -\frac{D}{m} - g\sin\gamma + \Omega_e^2 r\cos\varphi(\sin\gamma\cos\varphi - \cos\gamma\sin\varphi\sin\psi)$$

$$\dot{\gamma} = \frac{L\cos\sigma}{mV} - \frac{g\cos\gamma}{V} + \frac{V\cos\gamma}{r} + 2\Omega_e\cos\varphi\cos\psi$$
$$+\frac{\Omega_e^2 r}{V}\cos\varphi(\cos\gamma\cos\varphi + \sin\gamma\sin\varphi\sin\psi)$$

$$\dot{\psi} = \frac{L\sin\sigma}{mV\cos\gamma} - \frac{V}{r}\cos\gamma\cos\psi\tan\varphi + 2\Omega_e(\tan\gamma\cos\varphi\sin\psi - \sin\varphi)$$
$$-\frac{\Omega_e^2 r}{V\cos\gamma}\sin\varphi\cos\varphi\cos\psi$$

Point Mass Model for Spherical and Rotating Earth

Where

r – Radial Distance from Center of Earth

V – Earth Relative Velocity

Ω_e – Earth Angular Speed

γ – Flight Path Angle

σ – Velocity Roll / Bank Angle

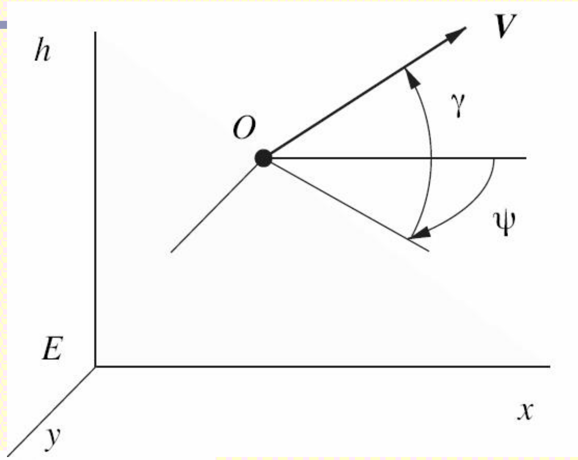
ψ – Velocity Yaw / Heading Angle

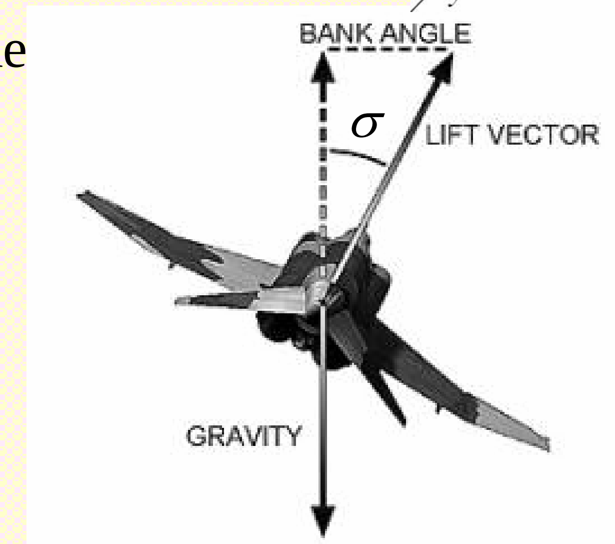
m – Mass of Vehicle

g – Acceleration due to Gravity

θ – Longitude

φ – Geocentric Latitude





Six – DOF Model

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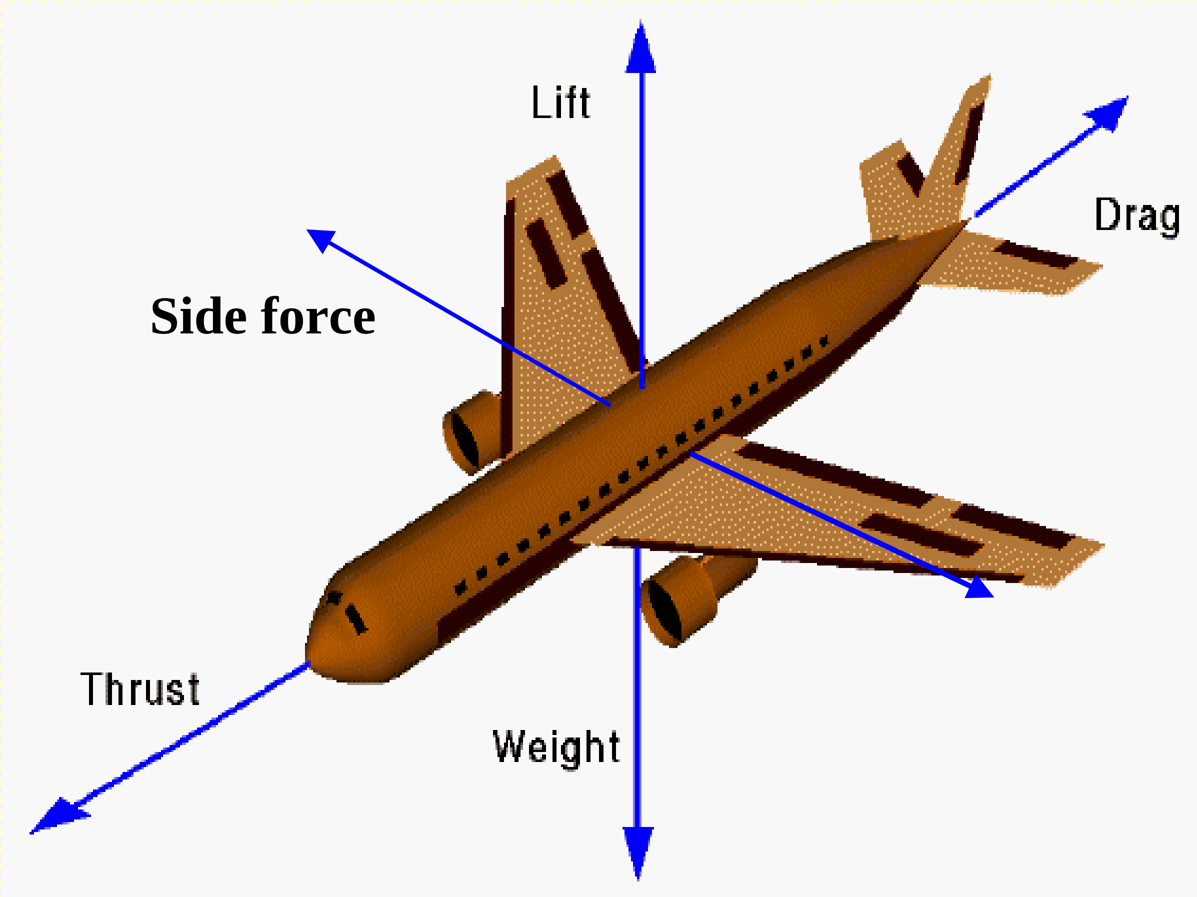
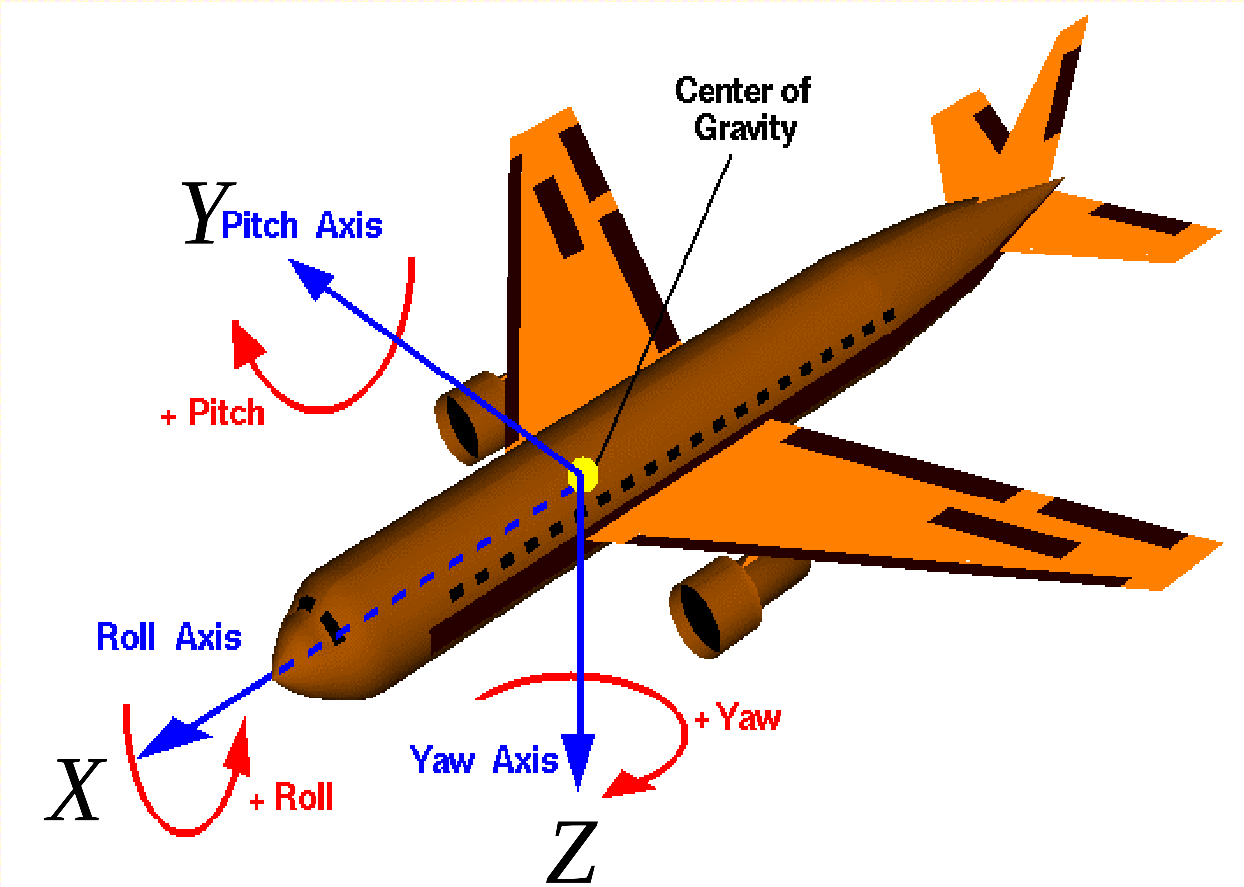
Basic Force Balance

- Weight
- Lift
- Drag
- Thrust
- <u>Side force</u>

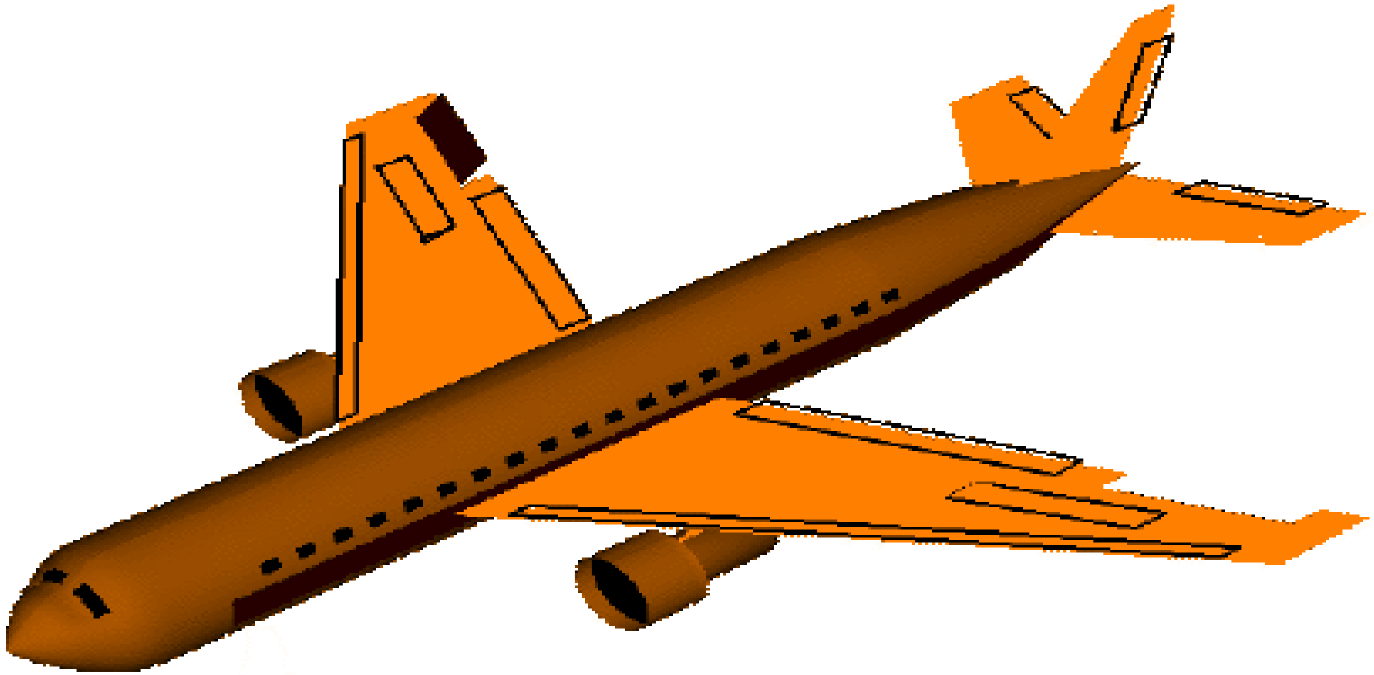


Basic Moment Balance

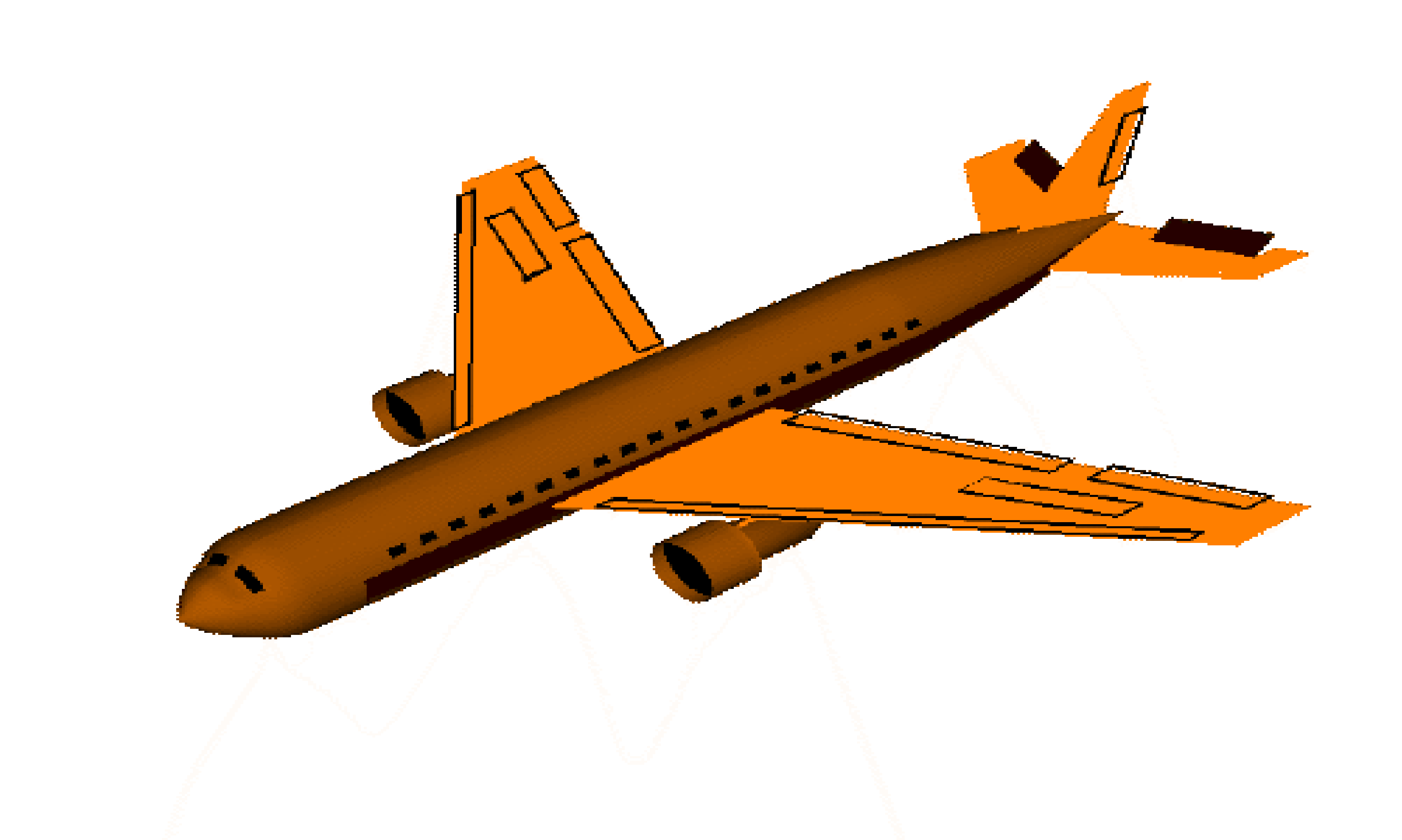
- Rolling
- Pitching
- Yawing



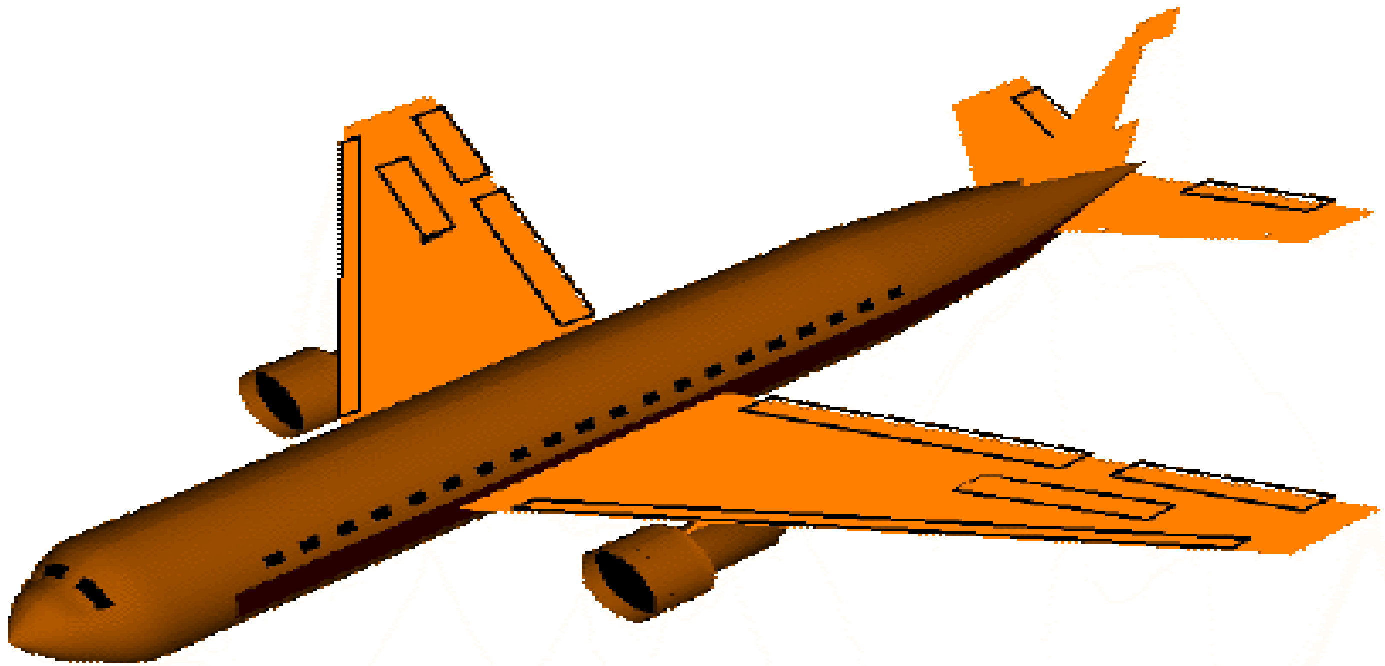
Control Action: Aileron ⇨ Roll



Control Action: Elevator ⇨ Pitch



Control Action: Rudder ⇨ Yaw

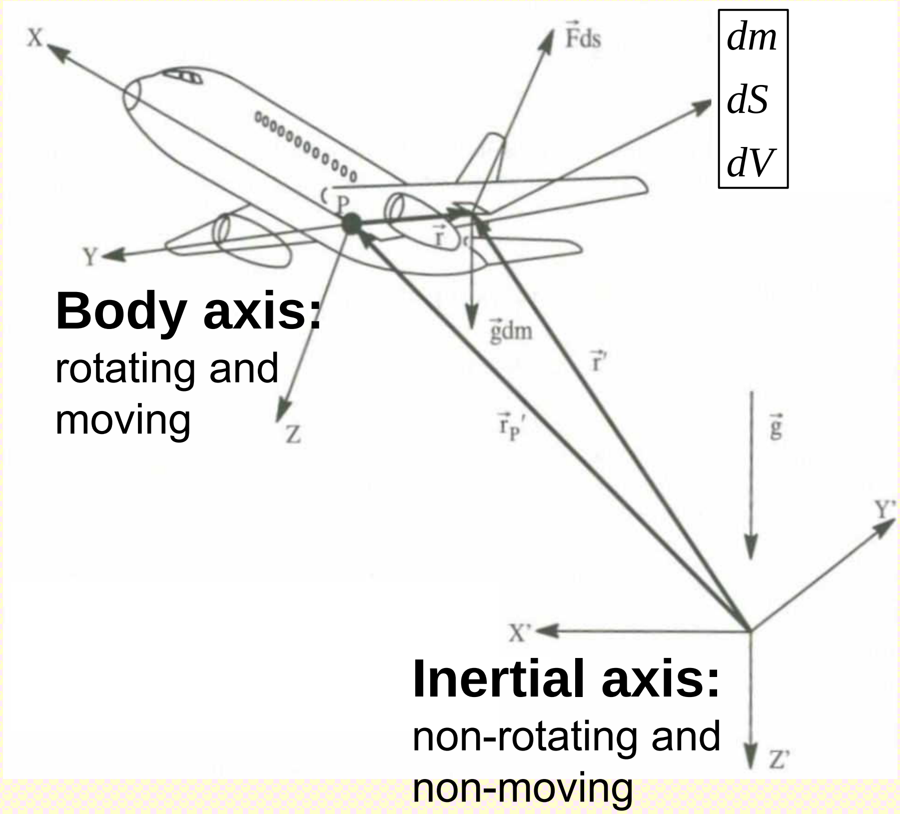


Six-DOF Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

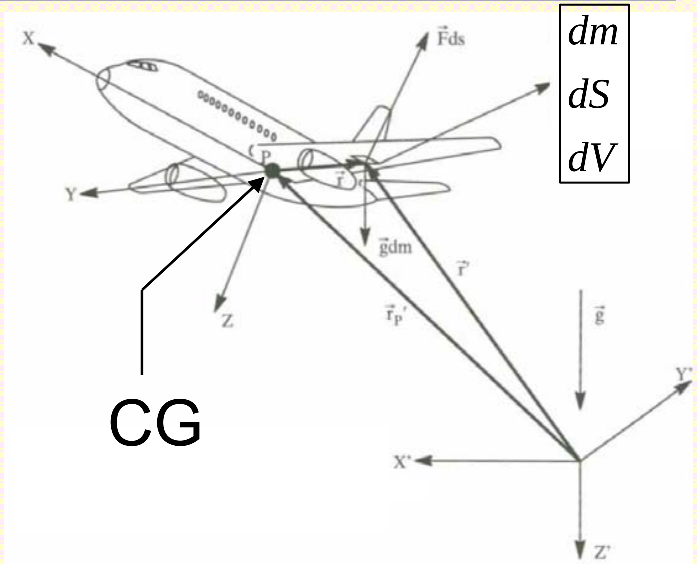
Assumptions:

- Flat earth (spherical and rotational effects are negligible)
- Rigid body (no relative motion of particles, no spinning rotors)
- Constant mass and mass distribution (no fuel burning, no fuel slosh, no passenger movement etc.)
- Uniform mass density
- Constant gravity



Six-DOF Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*



From Geometry: $\vec{r}\,' = \vec{r}_p\,' + \vec{r}$

P **is center of mass/gravity. Hence:**

$$\int_V \vec{r}\,\rho_A\,dV = 0$$

$$\int_V \left(\vec{r}\,' - \vec{r}_p\,'\right)\rho_A\,dV = 0$$

$$\int_V \vec{r}\,'\rho_A\,dV = \vec{r}_p\,'\int_V \rho_A\,dV = \vec{r}_p\,'\,m$$

$$\boxed{\vec{r}_p\,' = \frac{1}{m}\int_V \vec{r}\,'\rho_A\,dV}$$

Dynamic Equations

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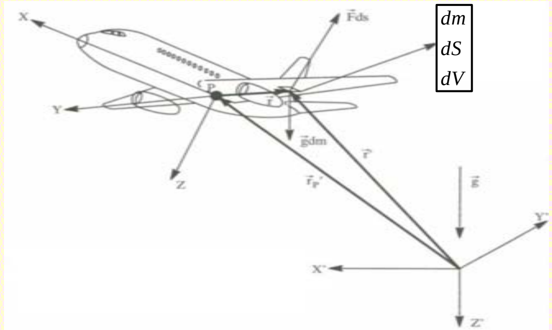
Force and Moment Equations

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

Newton's Second Law of Motion:
(valid in inertial reference frame)



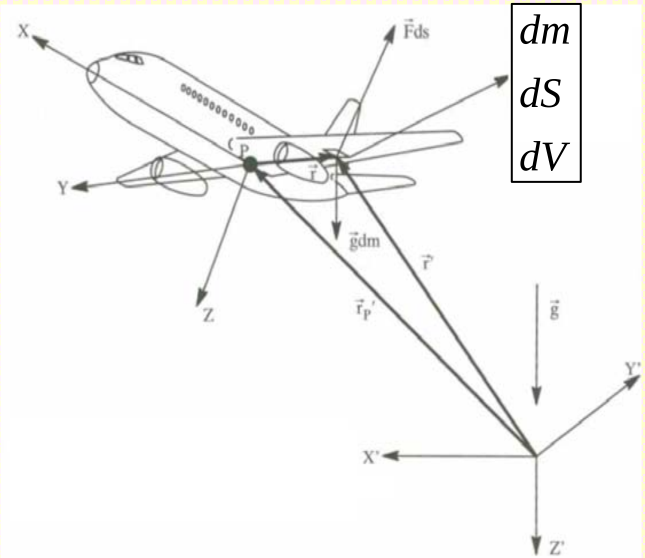
$$\underbrace{\frac{d}{dt}\left[\int_V \rho_A \left(\frac{d\vec{r}'}{dt}\right) dV\right]}_{\text{Linear momentum}} = \underbrace{\int_V \rho_A \vec{g}\, dV}_{\text{Gravity Force}} + \underbrace{\int_S \vec{F}\, ds}_{\text{Aerodynamic/Thrust Force}}$$

$$\underbrace{\frac{d}{dt}\left[\int_V \vec{r}' \times \rho_A \left(\frac{d\vec{r}'}{dt}\right) dV\right]}_{\text{Angular momentum}} = \underbrace{\int_V \vec{r}' \times \rho_A\, \vec{g}\, dV}_{\text{Gravity Moment}} + \underbrace{\int_S \vec{r}' \times \vec{F}\, ds}_{\text{Aerodynamic/Thrust Moment}}$$

Force Equation (inertial frame)

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\frac{d}{dt}\left[\int_V \rho_A \left(\frac{d\vec{r}'}{dt}\right) dV\right] = \frac{d}{dt}\left[\frac{d}{dt}\int_V \rho_A \left(\vec{r}_p' + \vec{r}\right) dV\right]$$

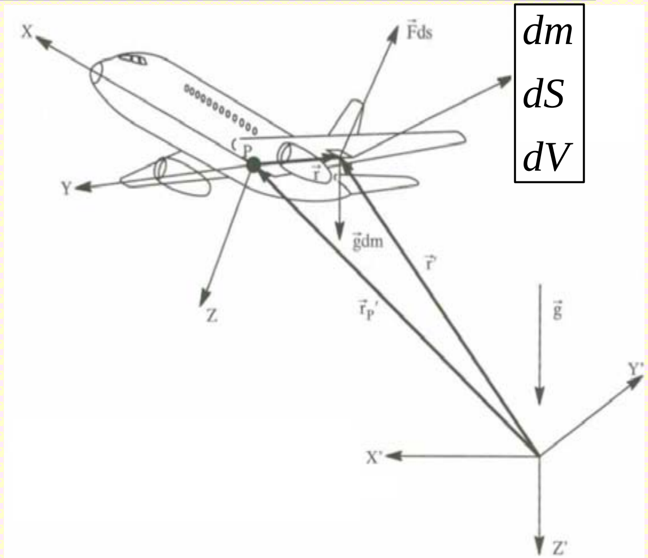


$$= \frac{d}{dt}\left[\frac{d}{dt}\left(\vec{r}_p' \underbrace{\int_V \rho_A dV}_{m} + \underbrace{\int_V \vec{r}\,\underbrace{\left(\rho_A\, dV\right)}_{dm}}_{=0}\right)\right]$$

$$= \frac{d}{dt}\left(\frac{d}{dt}\left(m\vec{r}_p'\right)\right) = m\frac{d}{dt}\underbrace{\left(\frac{d\vec{r}_p'}{dt}\right)}_{\vec{V}p} = m\frac{d\vec{V}p}{dt}$$

Force Equation (inertial frame)

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*



Moreover

$$\int_V \rho_A \vec{g}\, dV = \vec{g} \underbrace{\int_V \rho_A dV}_{m} = m\vec{g}$$

$$\int_S \vec{F}\, dS = \underbrace{\vec{X}}_{\text{Aerodynamic force}} + \underbrace{\vec{X}_T}_{\text{Thrust force}}$$

Hence

$$m\frac{d\vec{V}p}{dt} = \underbrace{m\vec{g} + \vec{X} + \vec{X}_T}_{\text{Applied in inertial frame}}$$

Force Equation (body frame)

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

- Thrust forces are typically applied in body frame
- Aerodynamic forces act on “wind frame”, which is close to body frame (they are same when $\alpha=\beta=0$)
- Body frame is a rotating frame. Hence it is NOT an inertial frame and the Newton’s laws are not applicable directly.

Force Equation (body frame)

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

<u>A Standard Result</u> :

$$\left.\frac{d\vec{A}}{dt}\right|_{\substack{\text{Equivalent}\\ \text{in inertial}\\ \text{frame}}} = \left.\frac{\partial\vec{A}}{\partial t}\right|_{\substack{\text{As seen}\\ \text{in rotating}\\ \text{frame}}} + \vec{\omega} \times \vec{A}$$

where $\vec{\omega}$: Angular velocity of rotating frame *wrt*. inertial frame.

$\vec{A}$: Any vector

Hence the force equation in body frame becomes :

$$m\left(\frac{\partial\vec{V}p}{\partial t}\right)_B + \left(\vec{\omega}\times\vec{V}p\right) = \underbrace{m\vec{g} + \vec{X} + \vec{X}_T}_{\substack{\text{Forced applied in}\\ \text{rotating (body) frame}}}$$

Force Equation (body frame)

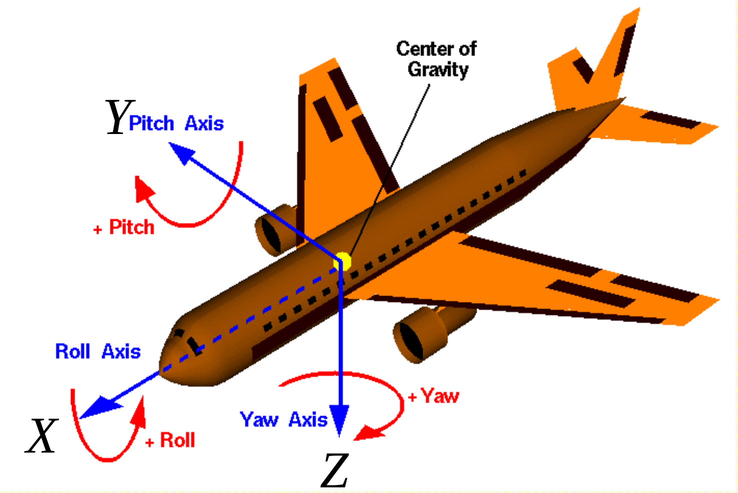
Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\vec{\omega} = \begin{bmatrix} P & Q & R \end{bmatrix}^T, \quad \left(\vec{V}p\right)_B = \begin{bmatrix} U & V & W \end{bmatrix}^T$$

$$\vec{X} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T, \quad \vec{X}_T = \begin{bmatrix} X_T & Y_T & Z_T \end{bmatrix}^T$$

$$\vec{g} = \begin{bmatrix} g_X & g_Y & g_Z \end{bmatrix}^T$$

$$\left(\vec{\omega} \times \vec{V}p\right) = \begin{vmatrix} i & j & k \\ P & Q & R \\ U & V & W \end{vmatrix}$$

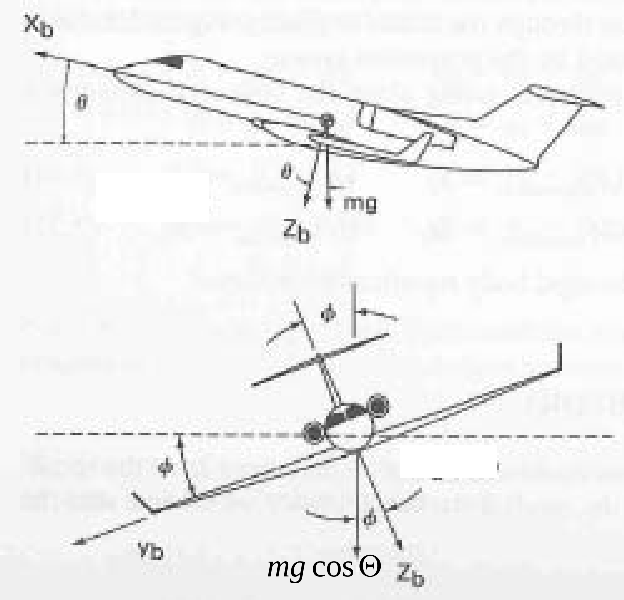


$$= i\left(QW - VR\right) - j\left(PW - UR\right) + k\left(PV - UQ\right)$$

Force Equation (body frame)

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$m\left(\dot{U} - VR + WQ\right) = mg_X + \left(X + X_T\right)$$
$$m\left(\dot{V} + UR - WP\right) = mg_Y + \left(Y + Y_T\right)$$
$$m\left(\dot{W} - UQ + VP\right) = mg_Z + \left(Z + Z_T\right)$$



where

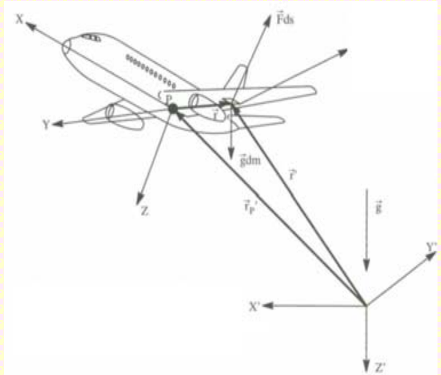
$$g_X = -g\sin\Theta$$
$$g_Y = g\cos\Theta\sin\Phi$$
$$g_Z = g\cos\Theta\cos\Phi$$

Note: The gravity components can also be Formally derived from Euler angle definitions.

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\underbrace{\frac{d}{dt}\left[\int_V \vec{r}\times\left(\frac{d\vec{r}}{dt}\right)\rho_A dV\right]}_{\text{Modified angular momentum (for rotating frame effect)}} = \underbrace{\int_S \vec{r}\times\vec{F}\,ds}_{\text{Applied moment in the body frame}}$$



$$= \underbrace{\vec{M}_A}_{\text{Aerodynamic moment}} + \underbrace{\vec{M}_T}_{\text{Thrust moment}}$$

Comment:

This expression can be derived from the earlier expression.

However, it is easier to visualize this equation directly in the body frame since, forces and moments act on the body frame and gravity force does not create any moment about CG.

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\frac{d}{dt}\int_V \vec{r}\times\left(\frac{d\vec{r}}{dt}\right)\rho_A dV = \int_V \frac{d}{dt}\left(\vec{r}\times\left(\frac{d\vec{r}}{dt}\right)\right)\rho_A\, dV$$

$$= \int_V \left(\underbrace{\left(\frac{d\vec{r}}{dt}\right)\times\left(\frac{d\vec{r}}{dt}\right)}_{=0} + \vec{r}\times\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right)\right)\rho_A\, dV$$

$$= \int_V \vec{r}\times\frac{d}{dt}\left(\underbrace{\dot{\vec{r}}}_{=0} + \vec{\omega}\times\vec{r}\right)\rho_A\, dV$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\frac{d}{dt}\int_V \vec{r}\times\left(\frac{d\vec{r}}{dt}\right)\rho_A dV = \int_V \vec{r}\times\left(\frac{\partial}{\partial t}(\vec{\omega}\times\vec{r})+\vec{\omega}\times(\vec{\omega}\times\vec{r})\right)\rho_A\, dV$$

$$= \int_V \vec{r}\times\left(\left(\dot{\vec{\omega}}\times\vec{r}+\vec{\omega}\times\underbrace{\dot{\vec{r}}}_{=0}\right)+\vec{\omega}\times(\vec{\omega}\times\vec{r})\right)\rho_A\, dV$$

$$= \int_V \vec{r}\times\left(\left(\dot{\vec{\omega}}\times\vec{r}\right)+\vec{\omega}\times(\vec{\omega}\times\vec{r})\right)\rho_A\, dV$$

Hence, the moment equation is:

$$\boxed{\int_V \vec{r}\times\left(\left(\dot{\vec{\omega}}\times\vec{r}\right)+\vec{\omega}\times(\vec{\omega}\times\vec{r})\right)\rho_A\, dV = \vec{M}_A+\vec{M}_T}$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

Standard Results :

$$\int_V \vec{r}\times\left(\dot{\vec{\omega}}\times\vec{r}\right)\rho_A\,dV = \int_V \left[\dot{\vec{\omega}}\left(\vec{r}\cdot\vec{r}\right)-\vec{r}\left(\vec{r}\cdot\dot{\vec{\omega}}\right)\right]\rho_A\,dV$$

$$\int_V \vec{r}\times\left(\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right)\rho_A\,dV = \int_V \vec{r}\times\left[\vec{\omega}\left(\vec{\omega}\cdot\vec{r}\right)-\vec{r}\left(\vec{\omega}\cdot\vec{\omega}\right)\right]\rho_A\,dV$$

$$= \int_V \vec{r}\times\vec{\omega}\left(\vec{\omega}\cdot\vec{r}\right)\rho_A\,dV$$

However,

$$\vec{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T \quad \text{(in body frame)}$$

$$\vec{\omega} = \begin{bmatrix} P & Q & R \end{bmatrix}^T$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\int_V \vec{r}\times\left(\dot{\vec{\omega}}\times\vec{r}\right)\rho_A\,dV = \int_V \left[\dot{\vec{\omega}}\left(\vec{r}\cdot\vec{r}\right)-\vec{r}\left(\vec{r}\cdot\dot{\vec{\omega}}\right)\right]\underbrace{\rho_A\,dV}_{dm}$$

$$= \dot{\vec{\omega}}\int_V r^2 dm - \int_V \vec{r}\left(\vec{r}\cdot\dot{\vec{\omega}}\right)dm$$

$$= \begin{bmatrix}\dot{P} & \dot{Q} & \dot{R}\end{bmatrix}^T \int_V \left(x^2+y^2+z^2\right)dm$$

$$-\begin{bmatrix}x & y & z\end{bmatrix}^T \int_V \left(x\dot{P}+y\dot{Q}+z\dot{R}\right)dm$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\int_V \vec{r}\times\left(\dot{\vec{\omega}}\times\vec{r}\right)\rho_A\, dV = \begin{bmatrix} \dot{P}\underbrace{\int_V \left(y^2+z^2\right)dm}_{I_{xx}} - \dot{Q}\underbrace{\int_V xy\, dm}_{I_{xy}} - \dot{R}\underbrace{\int_V xz\, dm}_{I_{xz}} \\ \dot{Q}\underbrace{\int_V \left(x^2+z^2\right)dm}_{I_{yy}} - \dot{P}\underbrace{\int_V yx\, dm}_{I_{yx}} - \dot{R}\underbrace{\int_V yz\, dm}_{I_{yz}} \\ \dot{R}\underbrace{\int_V \left(x^2+y^2\right)dm}_{I_{zz}} - \dot{P}\underbrace{\int_V zx\, dm}_{I_{zx}} - \dot{Q}\underbrace{\int_V zy\, dm}_{I_{zy}} \end{bmatrix}$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\int_V \vec{r} \times \left(\dot{\vec{\omega}} \times \vec{r} \right) \rho_A \, dV = \begin{bmatrix} I_{xx}\dot{P} - I_{xy}\dot{Q} - I_{xz}\dot{R} \\ I_{yy}\dot{Q} - I_{yx}\dot{P} - I_{yz}\dot{R} \\ I_{zz}\dot{R} - I_{zx}\dot{P} - I_{zy}\dot{Q} \end{bmatrix}$$

Similarly

$$\int_V \vec{r} \times \left(\vec{\omega} \times \left(\vec{\omega} \times \vec{r} \right) \right) \rho_A \, dV = \int_V \vec{r} \times \vec{\omega} \left(\vec{\omega} \cdot \vec{r} \right) \rho_A \, dV$$

$$= \begin{bmatrix} I_{xx}PR + I_{yz}\left(R^2 - Q^2\right) - I_{xz}PQ + \left(I_{zz} - I_{yy}\right)RQ \\ \left(I_{xx} - I_{zz}\right)PR + I_{xz}\left(P^2 - R^2\right) - I_{xy}QR + I_{yz}PQ \\ \left(I_{yy} - I_{xx}\right)PQ + I_{xy}\left(Q^2 - P^2\right) + I_{xz}QR - I_{yz}PR \end{bmatrix}$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

- **Assumption:** An airplane is symmetric about its XZ-plane

$$I_{xy} = I_{yz} = 0$$

- **Note:** Missiles and launch vehicles are typically symmetric about both XZ-plane as well as XY-plane

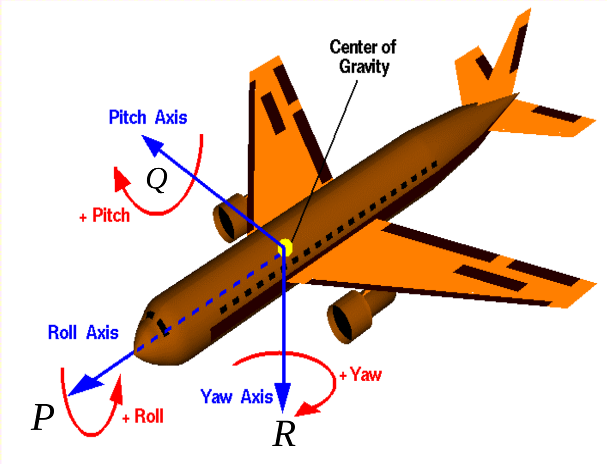
$$I_{xy} = I_{yz} = I_{zx} = 0$$

Moment Equation

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

Aero Moment: $\vec{M}_A = \begin{bmatrix} L & M & N \end{bmatrix}^T$

Thrust Moment: $\vec{M}_T = \begin{bmatrix} L_T & M_T & N_T \end{bmatrix}^T$



Hence, the moment equations are:

$$I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + \left(I_{zz} - I_{yy}\right)RQ = L + L_T$$

$$I_{yy}\dot{Q} + \left(I_{xx} - I_{zz}\right)PR + I_{xz}\left(P^2 - R^2\right) = M + M_T$$

$$I_{zz}\dot{R} - I_{xz}\dot{P} + \left(I_{yy} - I_{xx}\right)PQ + I_{xz}QR = N + N_T$$

Force and Moment Equations

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$m\left(\dot{U} - VR + WQ\right) = -mg\sin\Theta + \left(X + X_T\right)$$

$$m\left(\dot{V} + UR - WP\right) = mg\cos\Theta\sin\Phi + \left(Y + Y_T\right)$$

$$m\left(\dot{W} - UQ + VP\right) = mg\cos\Theta\cos\Phi + \left(Z + Z_T\right)$$

$$I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + \left(I_{zz} - I_{yy}\right)RQ = L + L_T$$

$$I_{yy}\dot{Q} + \left(I_{xx} - I_{zz}\right)PR + I_{xz}\left(P^2 - R^2\right) = M + M_T$$

$$I_{zz}\dot{R} - I_{xz}\dot{P} + \left(I_{yy} - I_{xx}\right)PQ + I_{xz}QR = N + N_T$$

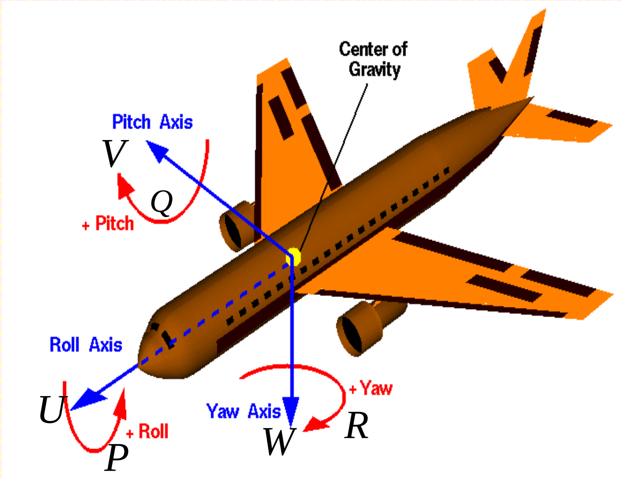
Force and Moment Equations

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\dot{U} = VR - WQ - g\sin\Theta + \frac{1}{m}\left(X + X_T\right)$$

$$\dot{V} = WP - UR + g\sin\Phi\cos\Theta + \frac{1}{m}\left(Y + Y_T\right)$$

$$\dot{W} = UQ - VP + g\cos\Phi\cos\Theta + \frac{1}{m}\left(Z + Z_T\right)$$



$$\dot{P} = c_1 QR + c_2 PQ + c_3\left(L + L_T\right) + c_4\left(N + N_T\right)$$

$$\dot{Q} = c_5 PR - c_6\left(P^2 - R^2\right) + c_7\left(M + M_T\right)$$

$$\dot{R} = c_8 PQ - c_2 QR + c_4\left(L + L_T\right) + c_9\left(N + N_T\right)$$

Force and Moment Equations

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

where

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_8 \\ c_9 \end{bmatrix} = \frac{1}{\left(I_{xx}I_{zz} - I_{xz}{}^2\right)} \begin{bmatrix} I_{zz}(I_{yy} - I_{zz}) - I_{xz}{}^2 \\ I_{xz}(I_{zz} + I_{xx} - I_{yy}) \\ I_{zz} \\ I_{yz} \\ I_{xx}(I_{xx} - I_{yy}) + I_{xz}{}^2 \\ I_{xx} \end{bmatrix}$$

$$c_5 = I_{zz} - I_{xx}) / I_{yy}$$

$$c_6 = I_{xz} / I_{yy}$$

$$c_7 = 1 / I_{yy}$$

Force and Moment Equations

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

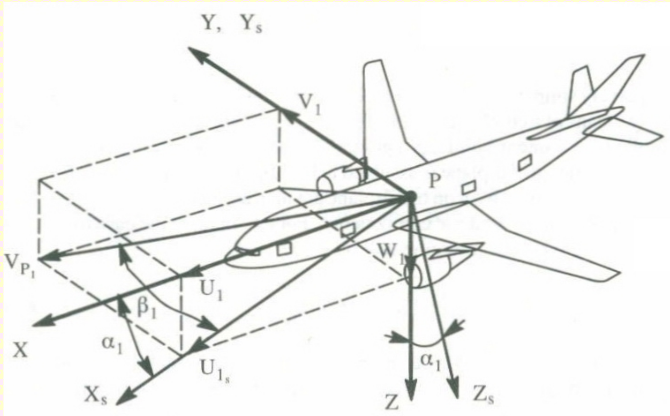
$$X_T = \sum_{i=1}^{N} T_i \cos\Phi_{T_i} \cos\Psi_{T_i} \qquad L_T = -\sum_{i=1}^{N}\left(T_i \cos\Phi_{T_i} \sin\Psi_{T_i}\right) z_{T_i} - \sum_{i=1}^{N}\left(T_i \sin\Phi_{T_i}\right) y_{T_i}$$

$$Y_T = \sum_{i=1}^{N} T_i \cos\Phi_{T_i} \sin\Psi_{T_i} \qquad M_T = \sum_{i=1}^{N}\left(T_i \cos\Phi_{T_i} \cos\Psi_{T_i}\right) z_{T_i} + \sum_{i=1}^{N}\left(T_i \sin\Phi_{T_i}\right) x_{T_i}$$

$$Z_T = -\sum_{i=1}^{N} T_i \sin\Phi_{T_i} \qquad N_T = -\sum_{i=1}^{N}\left(T_i \cos\Phi_{T_i} \cos\Psi_{T_i}\right) y_{T_i} + \sum_{i=1}^{N}\left(T_i \cos\Phi_{T_i} \sin\Psi_{T_i}\right) x_{T_i}$$

$$T_i = T_{i_{\max}} \cdot \sigma_{T_i}$$

$$\begin{bmatrix} X \\ Z \end{bmatrix} = T(\alpha)\begin{bmatrix} X_s \\ Z_s \end{bmatrix} = T(\alpha)(-\bar{q}S)\left(\begin{bmatrix} C_{D_O} & C_{D_\alpha} & C_{D_{i_h}} \\ C_{L_O} & C_{L_\alpha} & C_{L_{i_h}} \end{bmatrix}\begin{bmatrix} 1 \\ \alpha \\ i_h \end{bmatrix} + \begin{bmatrix} C_{D_{\delta_E}} \\ C_{L_{\delta_E}} \end{bmatrix}\delta_E\right)$$



$$\begin{bmatrix} L \\ N \end{bmatrix} = T(\alpha)\begin{bmatrix} L_s \\ N_s \end{bmatrix} = T(\alpha)\,\bar{q}Sb\left(\begin{bmatrix} C_{l_\beta} \\ C_{n_\beta} \end{bmatrix}\beta + \begin{bmatrix} C_{l_{\delta_A}} & C_{l_{\delta_R}} \\ C_{n_{\delta_A}} & C_{n_{\delta_R}} \end{bmatrix}\begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix}\right)$$

$$Y = \bar{q}S\, C_Y = \bar{q}S\left(C_{Y_\beta}\beta + \begin{bmatrix} C_{Y_{\delta_A}} & C_{Y_{\delta_R}} \end{bmatrix}\begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix}\right)$$

$$M = \bar{q}S\bar{c}\, C_m = \bar{q}S\bar{c}\begin{bmatrix} C_{m_o} & C_{m_\alpha} & C_{m_{i_h}} \end{bmatrix}\begin{bmatrix} 1 & \alpha & i_h \end{bmatrix}^T + C_{m_{\delta_E}}\delta_E$$

$$T(\alpha) \triangleq \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Comment

- We have derived only the set of "dynamic equations" of the Six-DOF model, which describe the effect of forces and moments.
- A set of "kinematic equations" are also needed to complete the Six-DOF model, which will be discussed in the next class.

Thanks for the Attention….!