Lecture – 5

Classical Control Overview – III

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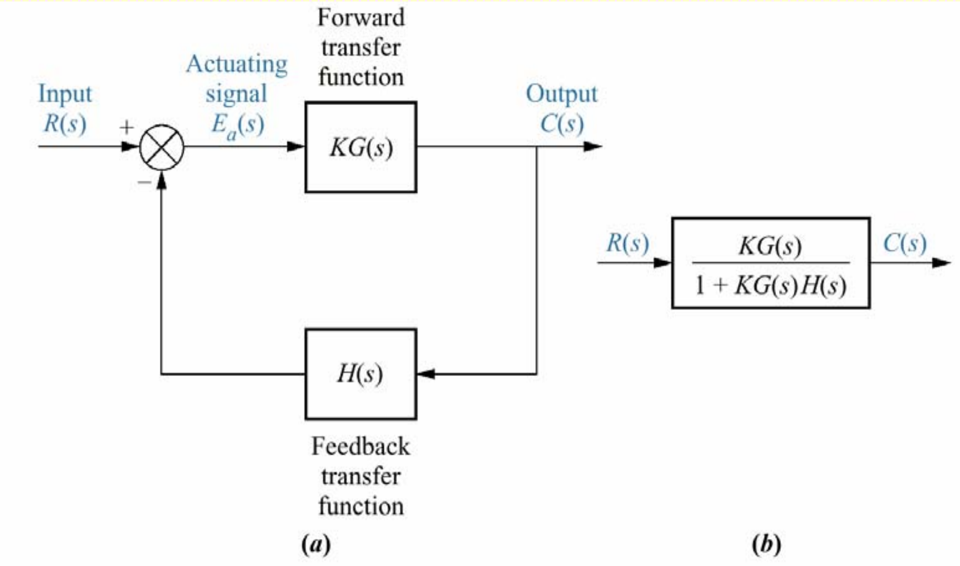
A Fundamental Problem in Control Systems

- Poles of open loop transfer function are easy to find and they do not change with gain variation either.
- Poles of closed loop transfer function, which dictate stability characteristics, are more difficult to find and change with gain



$$\text{open loop transfer function} = KG(s)H(s)$$

closed loop transfer function

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)}$$

let

$$G(s) = \frac{N_G(s)}{D_G(s)} \quad \text{and} \quad H(s) = \frac{N_H(s)}{D_H(s)}$$

$$T(s) = \frac{KN_G(s)D_H(s)}{D_G(s)D_H(s) + KN_G(s)N_H(s)}$$

Example

Let $G(s) = \dfrac{(s+1)}{s(s+2)}, \quad H(s) = \dfrac{(s+3)}{(s+4)}$

Then $KG(s)H(s) = \dfrac{K(s+1)(s+3)}{s(s+2)(s+4)}$

Poles of open loop Transfer Function $KG(s)H(s)$ *are* $(0, -2, -4)$

Now Closed Loop Transfer Function $T(s)$

$$T(s) = \frac{K(s+1)(s+4)}{s^3 + (6+K)s^2 + (8+4K)s + 3K}$$

– Poles of $T(s)$ are not immediately known; they depends on K as well.

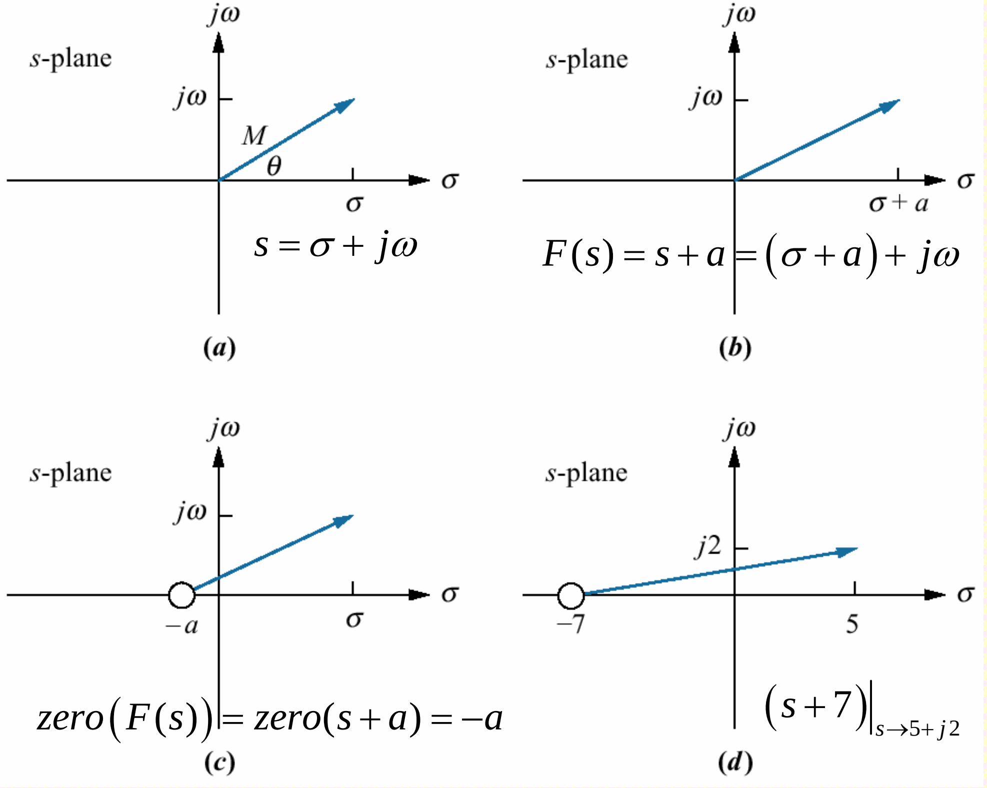
– System stability and transient response depends on poles of $T(s)$

– Root Locus gives vivid picture of the poles of $T(s)$ *as* K varies

Vector Representation of Complex Numbers and Complex Functions

Regular representation:

Alternate representation:



We can conclude that ***(s + a)*** is a complex number and can be represented by a vector drawn from the zero of the function **(- *a*)** to the point ‘***s’.***

Magnitude and Angle of Complex Functions

Let $F(s) = \dfrac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}, \quad m - number\ of\ zeros,\ n - number\ of\ poles$

The magnitude M of $F(s)$ at any point s is

$$M = \frac{\prod zero\ lengths}{\prod pole\ lengths} = \frac{\prod_{i=1}^{m}\left|(s+z_i)\right|}{\prod_{i=1}^{n}\left|(s+p_j)\right|}$$

The angle ,θ, of $F(s)$ at any point s is

$$\theta = \sum zero\ angles - \sum pole\ angles = \sum_{i=1}^{m}\angle(s+z_i) - \sum_{j=1}^{n}\angle(s+p_j)$$

Example

$$F(s) = \frac{(s+1)}{s(s+2)}$$

Vector originating at -1: $\sqrt{20}\ \angle 116.6^0$

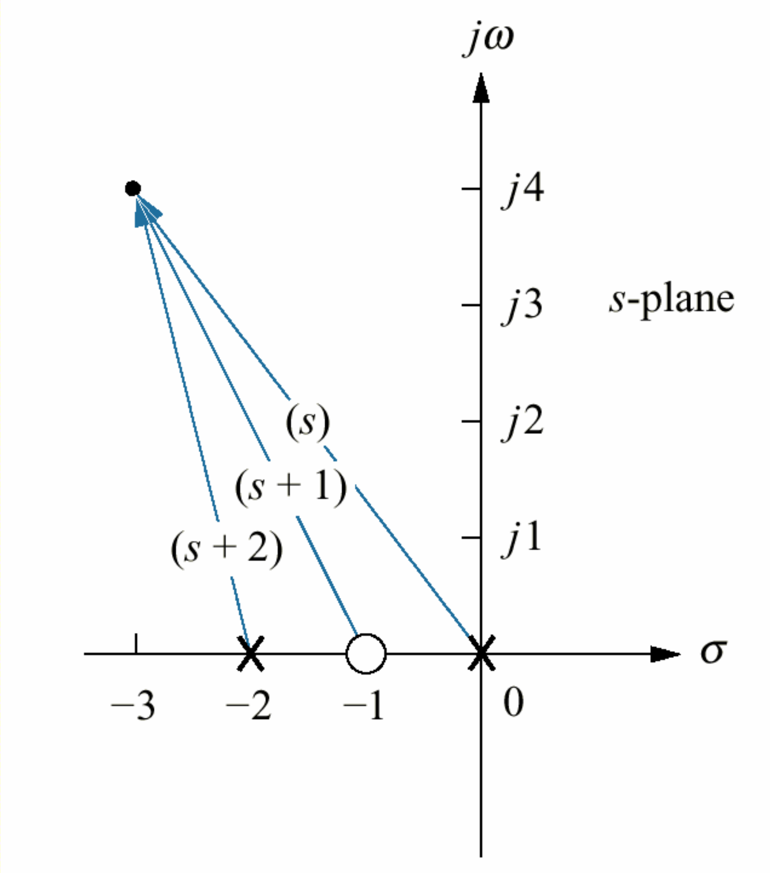
Vector originating at 0: $5\ \angle 126.9^0$

Vector originating at -2: $\sqrt{17}\ \angle 104.0^0$

$\therefore\ M\angle\theta$ of $F(s)$

$$= \frac{\sqrt{20}}{5\sqrt{17}} \quad \angle 116.6^0 - 126.9^0 - 104.0^0$$

$$= 0.217 \quad \angle -114.3^0$$



Root Locus Analysis

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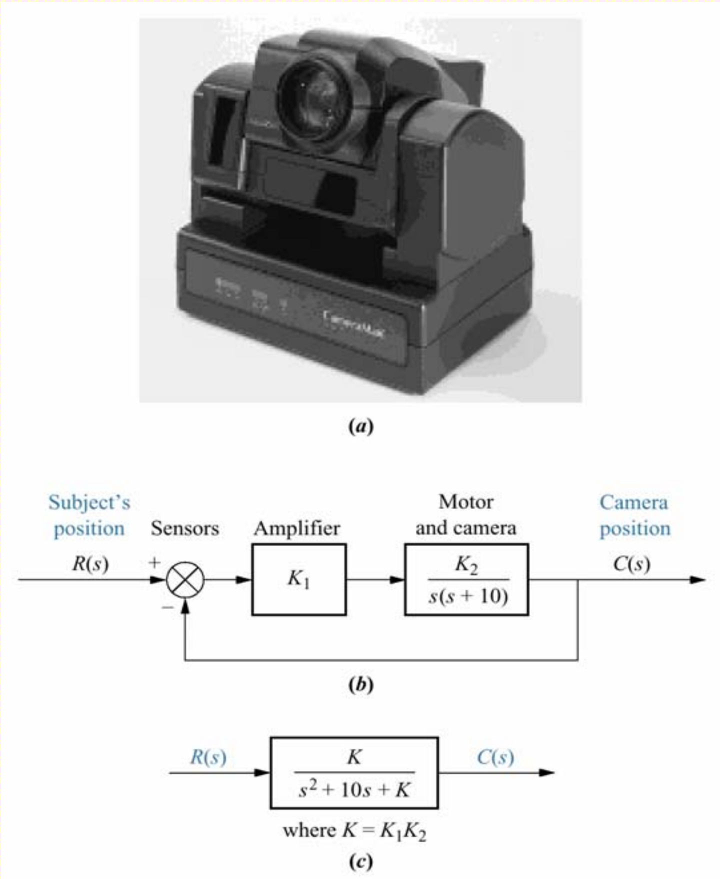
Facts

- Introduced by W. R. Evans (1948)
- Graphical representation of the ***closed loop poles*** in *s*-plane as the system parameter (typically controller gain) is varied.
- Gives a graphic representation of a system's stability characteristics
- Contains both qualitative and quantitative information
- Holds good for higher order systems

A Motivating Example



Ref: N. S. Nise:
Control Systems Engineering,
4th Ed., Wiley, 2004

Pole location as a function of gain for the system.

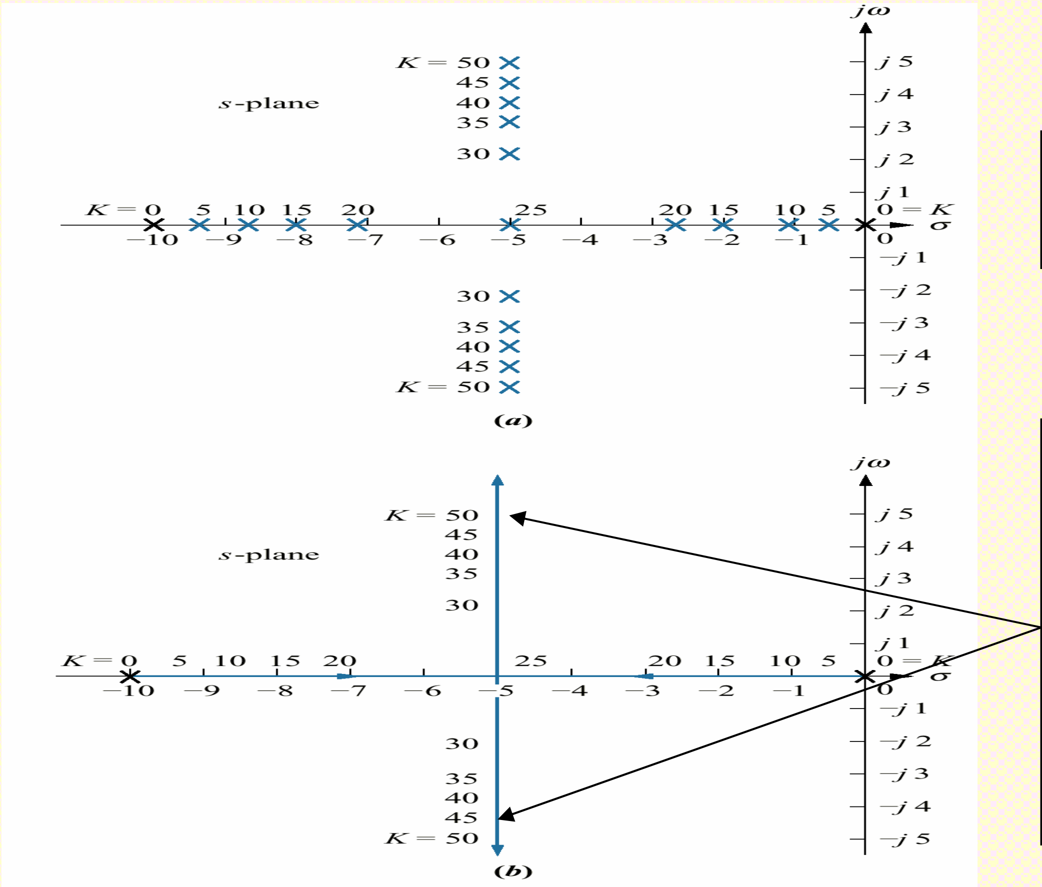
K	Pole 1	Pole 2
0	-10	0
5	-9.47	-0.53
10	-8.87	-1.13
15	-8.16	-1.84
20	-7.24	-2.76
25	-5	-5
30	$-5 + j2.24$	$-5 - j2.24$
35	$-5 + j3.16$	$-5 - j3.16$
40	$-5 + j3.87$	$-5 - j3.87$
45	$-5 + j4.47$	$-5 - j4.47$
50	$-5 + j5$	$-5 - j5$

over damped (K = 0 to 25)

under damped (K = 30 to 50)

Note: Typically K > 0 for negative feedback systems

A Motivating Example



Poles are always negative:
•system is always stable

Real parts are same:
- same settling time

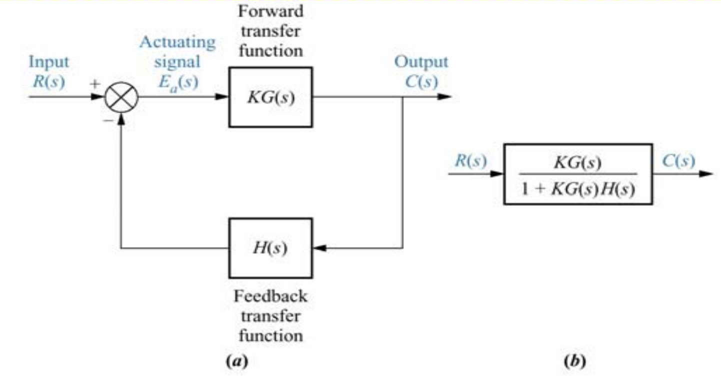
As gain increases,
- damping ratio decreases
- %age overshoot increases

Properties of Root Locus

The closed loop Transfer Function

$$T(s) = \frac{KG(s)}{1+KG(s)H(s)}$$

Characteristics equation: $1+KG(s)H(s)=0$



Closed loop poles are solution of characteristics equation.

However, $1+KG(s)H(s)=0$ is a complex quantity. Hence, it can be expressed as

$$KG(s)H(s) = -1 = 1\angle(2k+1)180^0 \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\ldots$$

The above condition (Evan's condition) can be written as

$$\left|KG(s)H(s)\right| = 1 \quad \text{(Magnitude criterion)}$$

$$\angle KG(s)H(s) = (2k+1)180^0 \quad \text{(Angle criterion)}$$

Example

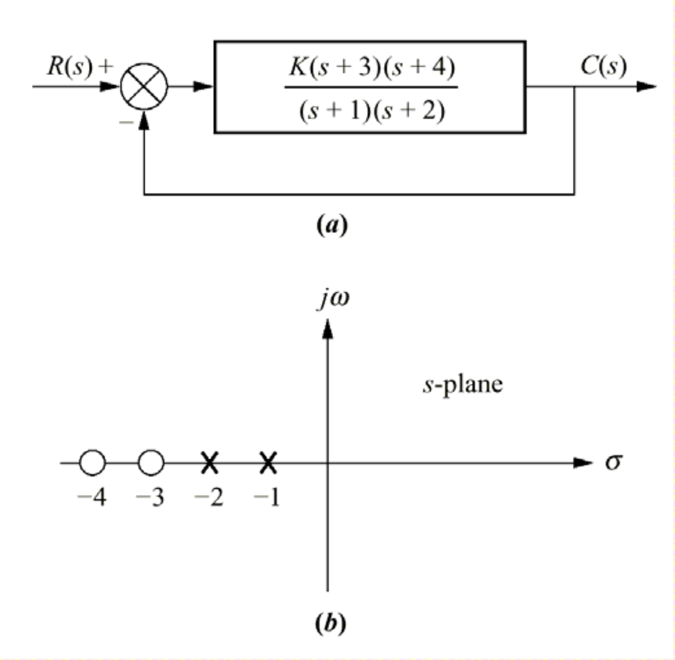
Fact : If the angle of a complex number is an odd multiple of 180^0 for an open loop transfer function, then it is a pole of the closed loop system with

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{|G(s)||H(s)|}$$

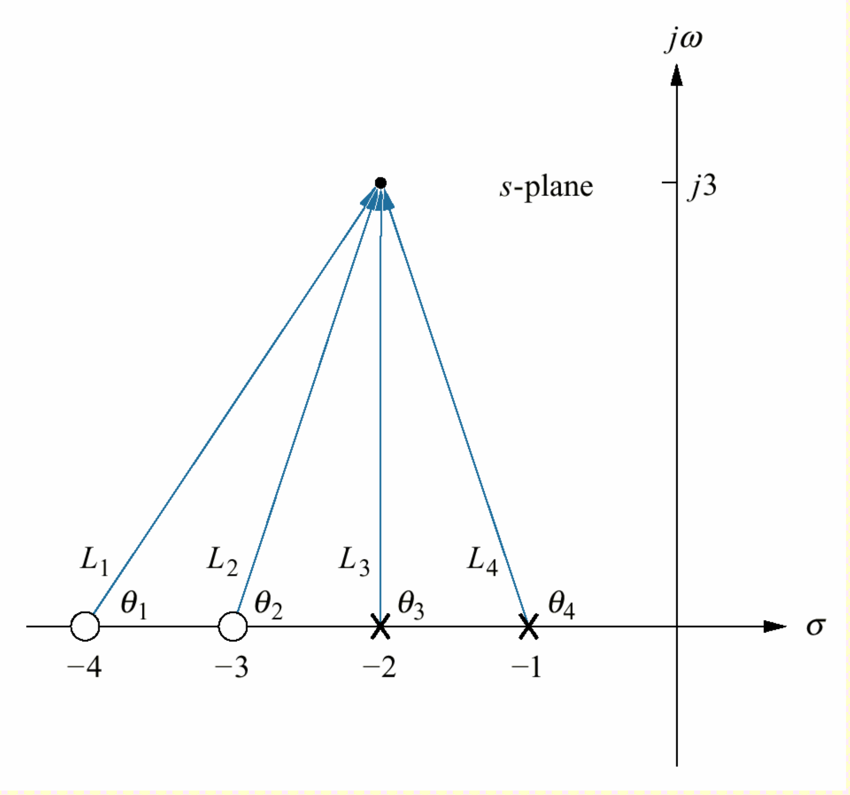
Example: Let us consider a system as in the Figure and consider two points:

$$P_1 : -2 + j3$$

$$P_2 : -2 + j\left(\frac{\sqrt{2}}{2}\right)$$



Example



<u>For P_1:</u> $\sum \theta_i = \theta_1 + \theta_2 - \theta_3 - \theta_4 = 56.31 + 71.57 - 90 - 108.43$

$$= -70.55^0 \neq (2k+1)180^0$$

Therefore $-2 + j3$ is not a point on root locus

<u>For P_2:</u> $\sum \theta_i = 180^0$

Hence $-2 + j\left(\dfrac{\sqrt{2}}{2}\right)$ is on root locus for some value of K.

Moreover, $K = \dfrac{L_3 L_4}{L_1 L_2} = \dfrac{\dfrac{\sqrt{2}}{2} \times (1.22)}{(2.12) \times (1.22)} = 0.33$

Summary

Given the poles and zeros of the open loop transfer function $KG(s)H(s)$, a point in the s-plane is on the root locus for a particular value of gain K, if the angles of the zeros minus the angles of the poles add up to $(2k+1)180^0$.

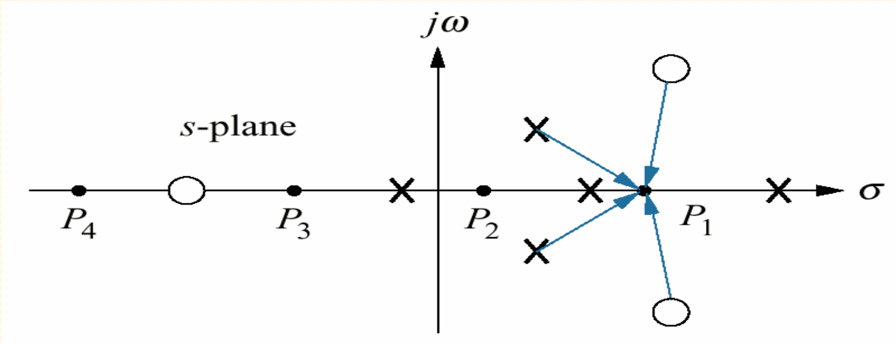
Furthermore, the gain K at that point can be found by dividing the product of the lengths of the poles by the product of the lengths of the zeros.

Sketching the Root Locus: Basic Five Rules

1. **Number of Branches:** The number of branches of the root locus equals the number of closed loop poles (since each pole should move as the gain varies).

2. **Symmetricity:** A root locus is always symmetric about the real axis, since complex poles must always appear in conjugate pairs.

3. **Real-axis segments:** On the real axis, for K > 0, the root locus exists to the left of an odd number of real-axis finite open-loop poles and zeros.



Sketching the Root Locus: Basic Five Rules

4. **Starting and ending points:** The root locus begins at the finite and infinite poles of *G(s)H(s)* and ends at the finite and infinite zeros of *G(s)H(s)*

5. **Behavior at Infinity:** The root locus approaches straight lines as asymptotes as the locus approaches infinity

The equation of asymptotes is given by the real-axis intercept σ_a and angle θ_a as follows:

$$\sigma_a = \frac{\sum finite\ poles - \sum finite\ zeros}{No.of\ finite\ poles - No.of\ finite\ zeros}$$

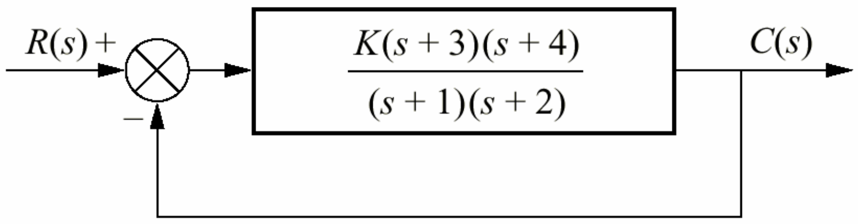
$$\theta_a = \frac{(2k+1)\Pi}{No.of\ finite\ poles - No.of\ finite\ zeros}$$

$where\ k = 0, \pm 1, \pm 2, \pm 3, ...$

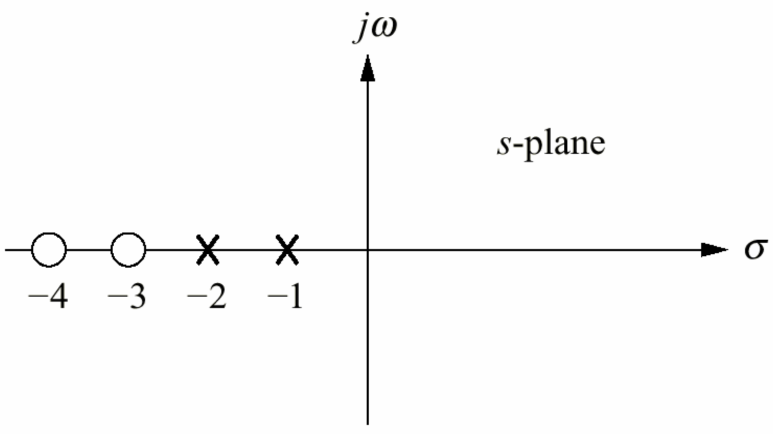
and the angle is given in radians wrt. positive extension of real axis

For **additional rules**, refer to:
N. S. Nise: Control Systems Engineering, 4th Ed., Wiley, 2004.

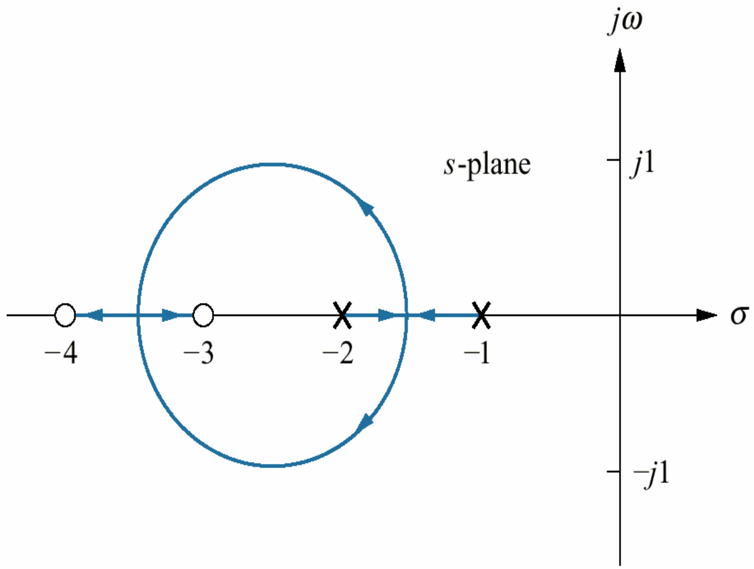
Example



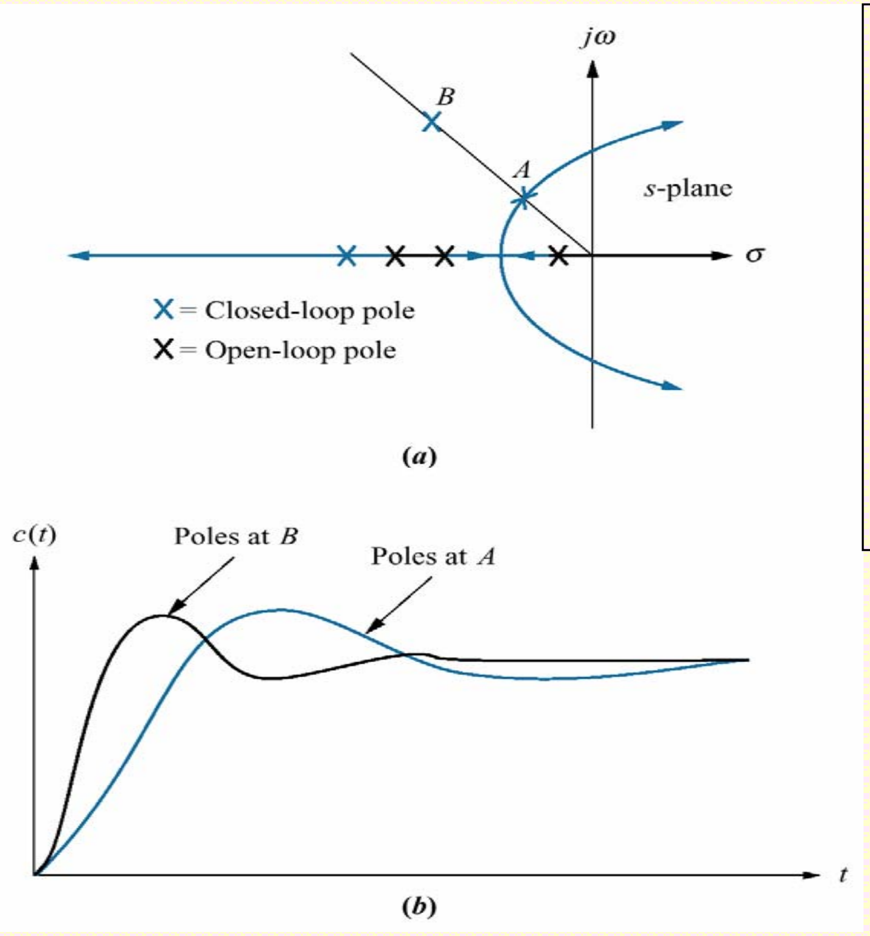
(a)



(b)



Control Tuning via Gain Adjustment using Root Locus



Sample root locus:

A. Possible design point via gain adjustment

B. Desired design point that cannot be met via simple gain adjustment (needs dynamic compensators)

PID Control Design

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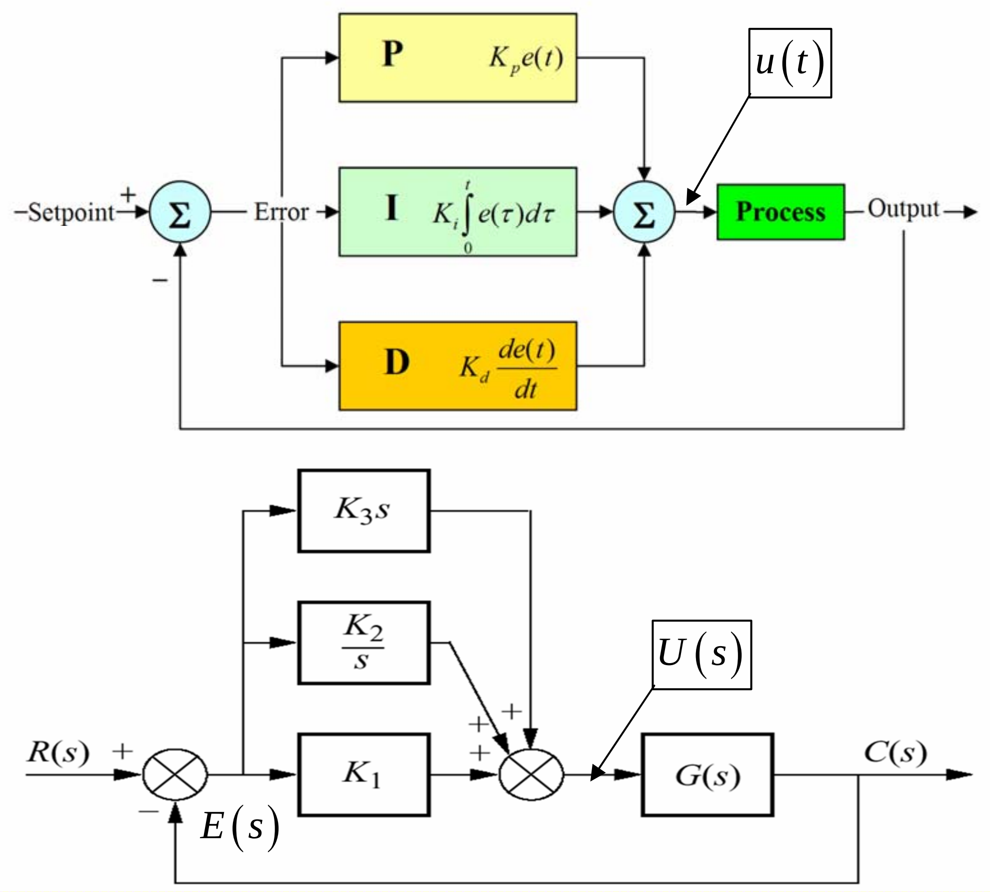
Introduction

Ref : *en.wikipedia.org/wiki/PID_controller*

- The PID controller involves three components:
  - **Proportional feedback**
  - **Integral feedback**
  - **Derivative feedback**
- By **tuning the three components (gains)**, a suitable control action is generated that leads to desirable closed loop response of the output.



Ref: N. S. Nise: Control Systems Engineering, 4th Ed., Wiley, 2004.

Philosophy of PID Design

- The **proportional** component determines the reaction to the **current value of the output error**. It serves as a “all pass” block.
- The **integral** component determines the reaction based on the **integral (sum) of recent errors**. In a way, it accounts for the history of the error and serves as a “low pass” block.
- The **derivative** component determines the reaction based on the **rate of change of the error.** In a way, it accounts for the future value of the error and serves as a “high pass” block.

PID Controller

The final form of the PID algorithm is:

$$u(t) = K_p e(t) + K_i \int_o^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

$$\frac{U(s)}{E(s)} = K_p + K_i / s + K_d s$$

The tuning parameters are:

Proportional gain, K_p

Integral gain, $K_i = K_p / T_i$

Derivative gain, $K_d = K_p T_d$

Effect of Proportional Term

- **<u>Proportional Term</u>**:

$$P_{out} = K_p e(t)$$

P_{out}: Proportional term of output

K_p: Proportional gain (a tuning parameter)

- If the gain K_p is **low**, then control action may be too small when responding to system disturbances. Hence, it need not lead to desirable performance.
- If the proportional gain K_p is **too high**, the system can become unstable. It may also lead to noise amplification

Effect of Integral Term

Integral term :

$$I_{out} = K_i \int_0^t e(\tau)d\tau$$

I_{out}: Integral term of output

K_i: Integral gain, a tuning parameter

The integral term (when added to the proportional term) accelerates the movement of the process towards setpoint and eliminates the residual steady-state error that occurs with a proportional only controller.

Effect of Integral Term

Destabilizing effects of the Integral term :

It can be seen that adding an integral term to a pure proportional term increases the gain by a factor of

$$\left| 1 + \frac{1}{j\omega T_{\mathrm{i}}} \right| = \sqrt{1 + \frac{1}{\omega^2 T_i^2}} > 1, \text{ for all } \omega.$$

and simultaneously increases the phase-lag since

$$\angle \left(1 + \frac{1}{j\omega T_{\mathrm{i}}} \right) = \tan^{-1}\left(\frac{-1}{\omega T_i} \right) < 0 \text{ for all } \omega.$$

Because of this, both the gain margin (GM) and phase margin (PM) are reduced, and the closed-loop system becomes more oscillatory and potentially unstable.

Ref : Li, Y. , Ang, K.H. and Chong, G.C.Y. (2006)
PID Control System Analysis and Design.
IEEE Control Systems Magazine 26(1):pp. 32-41.

Effect of Integral Term

- **Integrator Windup:**

  If the actuator that realizes control action has saturated and if it is neglected, this causes low frequency oscillations and leads to instability.

  Remedies:

  Automatic Resetting, Explicit Anti-windup.

Effect of Derivative Term

Derivative term :

$$D_{out} = K_d \frac{d}{dt} e(t)$$

K_d: Derivative gain (a tuning parameter)

- The derivative term speeds up the transient behaviour. In general, it has negligible effect on the steady state performance (for step inputs, the effect on steady state response is zero).
- Differentiation of a signal amplifies noise. Hence, this term in the controller is highly sensitive to noise in the error term! Because of this, the derivative compensation should be used with care.

Effect of Derivative Term

Adding a derivative term to pure proportional term reduces the phase lag by

$$\angle\left(1+j\omega T_d\right)=\tan^{-1}\left(\frac{\omega T_d}{1}\right)\in\left[0,\pi/2\right]\ \text{for all}\,\omega$$

which tends to increase the Phase Margin.

In the meantime, however, the gain increases by a factor of

$$\left|1+j\omega T_d\right|=\sqrt{1+\omega^2 T_d^2}>1,\ \text{for all}\ \omega.$$

which decreases the Gain Margin.

Hence the overall stability may be improved or degraded.

Additional requirement : Low pass filter, Set-point filter, Pre-filter etc.

Effect of Increasing Gains on Output Performance

Parameter	**Rise time**	**Overshoot**	**Settling time**	**Error at equilibrium**
K_p	Decreases	Increases	Small change	Decreases
K_i	Decreases	Increases	Increases	Eliminated
K_d	Indefinite (can either decrease or increase)	Decreases	Decreases	No Effect

Tuning of PID Design

Method	Advantages	Disadvantages
Manual Tuning	No math required. Largely trial-and-error approach (based on general observations)	Requires experienced personnel
Cohen-Coon	Results in good tuning in general	Offline method. Good for first-order processes
Ziegler–Nichols	Proven online method.	Very aggressive tuning, May upset some inherent advantages of the process
Software Tools	Online or offline methods.	Some cost and training of personnel is involved

Ziegler–Nichols Method for Gain Tuning

The K_i and K_d gains are first set to zero. The P gain is increased until it reaches the critical gain, K_c, at which the output of the loop starts to oscillate. Next, K_c and the oscillation period P_c are used to set the gains as per the following rule.

Control Type	K_p	K_i	K_d
P	$0.50K_c$	-	-
PI	$0.45K_c$	$1.2K_p / P_c$	-
PID	$0.60K_c$	$2K_p / P_c$	$K_p P_c / 8$

Note: The constants used may vary depending on the application.

Limitations of PID control

- It is a SISO design approach and hence can effectively handle only such system
- System should behave in a fairly linear manner. Hence, it is valid in close proximity of an operating point (about which the linearized system is valid)
- Does not take into account the limitations of the actuators

<u>Techniques to overcome</u>:

- Gain scheduling, Cascading controllers, Filters in loop etc.

Thanks for the Attention….!