Lecture – 30

Applications Linear Control Design Techniques in Aircraft Control – II

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Topics

- Brief Review of Modern Control Design for Linear Systems
- Automatic Flight Control Systems: Modern (Time Domain) Designs

Brief Review of Modern Control Design for Linear Systems

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State Space Representation for Dynamical Systems

- **Nonlinear System**

$$\dot{X} = f(X, U) \qquad X \in R^n, U \in R^m$$

$$Y = h(X, U) \qquad Y \in R^p$$

- **Linear System**

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

A - System matrix- *n x n*

B - Input matrix- *n x m*

C - Output matrix- *p x n*

D - Feed forward matrix – *p x m*

Stability of Linear System

$$\dot{X} = AX \quad , X(0) = X_0$$

Question:

Can we conclude about nature of the solution, without solving the system model?

Answer: YES!

Definition: Eigenvalues of *A* : "Poles" of the system!

The nature of the solution is governed only by the locations of its poles

Controllability

Result: If the rank of $C_B \triangleq \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$ is n, then the system is controllable.

Example:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

$$C_B = \left[\begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} 2 & -2 \\ 1 & -2 \end{bmatrix}$$

$rank\left(C_B\right) = 2 \quad \therefore$ The system is controllable.

Observability

Result: If the rank of $O_B \triangleq \begin{bmatrix} C^T & A^T C^T & \cdots & \left(A^T\right)^{n-1} C^T \end{bmatrix}$ is n, then the system is observable.

Example:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$O_B = \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$rank\left(O_B\right) = 1 \neq 2 \quad \therefore$ The system is NOT observable.

Closed Loop System Dynamics

$$\dot{X} = AX + BU$$

The control vector U is designed in the following state feedback form

$$U = -KX$$

This leads to the following closed loop system

$$\dot{X} = (A - BK)X = A_{CL}X$$

where $A_{CL} \triangleq (A - BK)$

Pole Placement Control Design

Objective:

The closed loop poles should lie at $\mu_1, \ldots, \mu_n$, which are their 'desired locations'.

Difference from classical approach:

Not only the "dominant poles", but "all poles" are forced to lie at specific desired locations.

Necessary and sufficient condition:

The system is completely state controllable.

Philosophy of Pole Placement Control Design

The gain matrix K is designed in such a way that

$$\left| sI - \left(A - BK \right) \right| = \left(s - \mu_1 \right)\left(s - \mu_2 \right) \quad \cdots \quad \left(s - \mu_n \right)$$

where $\mu_1, \cdots, \mu_n$ are the desired pole locations.

Pole Placement Design Steps: Method 1 (low order systems, $n \le 3$)

- Check controllability
- Define $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$
- Substitute this gain in the desired characteristic polynomial equation

$$\left| sI - A + BK \right| = (s - \mu_1) \cdots (s - \mu_n)$$

- Solve for k_1, k_2, k_3 by equating the like powers on both sides

Pole Placement Design: Summary of Method 2 (Bass-Gura Approach)

- Check the controllability condition
- Form the characteristic polynomial for *A*

  $$|sI - A| = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n$$

  find a_i's
- Find the Transformation matrix $T = M\,W$
- Write the desired characteristic polynomial

  $$(s-\mu_1)\cdots(s-\mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_n$$

  and determine the α_i's
- The required state feedback gain matrix is

  $$K = [(\alpha_n - a_n) \quad (\alpha_{n-1} - a_{n-1}) \quad \cdots \quad (\alpha_1 - a_1)]\, T^{-1}$$

Pole Placement Design Steps: Method 3 (Ackermann's formula)

For an arbitrary positive integer *n* (number of states) *Ackermann's formula* for the state feedback gain matrix *K* is given by

$$K = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} B & AB & A^2 B \cdots\cdots & A^{n-1} B \end{bmatrix}^{-1} \phi(A)$$

where

$$\phi(A) = A^n + \alpha_1 A^{n-1} + \cdots\cdots\cdots + \alpha_{n-1} A + \alpha_n I$$

and α_i's are the coefficients of the desired characteristic polynomial

LQR Design: Problem Statement

- Performance Index (to minimize):

$$J = \frac{1}{2}\left(X_f^T S_f X_f \right) + \frac{1}{2}\int_{t_0}^{t_f} \left(X^T Q X + U^T R U \right) dt$$

- Path Constraint: $\dot{X} = A X + B U$

- Boundary Conditions: $X(0) = X_0$: Specified

  t_f : Fixed, $X(t_f)$: Free

LQR Design:
Necessary Conditions of Optimality

- Terminal penalty: $\varphi\left(X_f\right) = \dfrac{1}{2}\left(X_f^T S_f X_f\right)$
- Hamiltonian: $H = \dfrac{1}{2}\left(X^T Q X + U^T R U\right) + \lambda^T \left(AX + BU\right)$
- State Equation: $\dot{X} = AX + BU$
- Costate Equation: $\dot{\lambda} = -\left(\partial H / \partial X\right) = -\left(QX + A^T \lambda\right)$
- Optimal Control Eq.: $\left(\partial H / \partial U\right) = 0 \quad \Rightarrow \quad U = -R^{-1} B^T \lambda$
- Boundary Condition: $\lambda_f = \left(\partial \varphi / \partial X_f\right) = S_f X_f$

LQR Design: Riccati Equation

- Riccati equation

$$\dot{P} + PA + A^T P - PBR^{-1}B^T P + Q = 0$$

- Boundary condition

$$P(t_f)X_f = S_f X_f \quad (X_f \text{ is free})$$

$$P(t_f) = S_f$$

LQR Design: Solution Procedure

- Use the boundary condition $P(t_f) = S_f$ and integrate the Riccati Equation backwards from t_f to t_0
- Store the solution history for the Riccati matrix
- Compute the optimal control online

$$U = -\left(R^{-1} B^T P\right) X = -K X$$

LQR Design: Infinite Time Regulator Problem

Theorem (By Kalman)

As $t_f \to \infty$, for constant Q and R matrices, $\dot{P} \to 0 \quad \forall t$

Algebraic Riccati Equation (ARE)

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

Note:

- ARE is still a nonlinear equation for the Riccati matrix. It is not straightforward to solve. However, efficient numerical methods are now available.
- A positive definite solution for the Riccati matrix is needed to obtain a stabilizing controller.

Summary of LQR Design: Infinite Time Regulator Problem

Problem:

State equation: $\dot{X} = AX + BU$

Cost function: $J = \frac{1}{2}\int_0^{\infty}\left(X^T Q X + U^T R U\right) dt$

Solution:

Solve the ARE: $PA + A^T P - PBR^{-1}B^T P + Q = 0$

Compute the control: $U = -\left(R^{-1}B^T P\right) X = -K X$

Automatic Flight Control Systems: Time Domain Designs

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Applications of Automatic Flight Control Systems

- Stability Augmentation Systems
  - Stability enhancement
  - Handling quality enhancement
- Cruise Control Systems
  - Attitude control (to maintain pitch, roll and heading)
  - Altitude hold (to maintain a desired altitude)
  - Speed control (to maintain constant speed or Mach no.)
- Landing Aids
  - Alignment control (to align wrt. runway centre line)
  - Glideslope control
  - Flare control

Stability Augmentation Systems

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Stability Augmentation System (SAS)

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Inherent stability of an airplane depends on the aerodynamic stability derivatives.
- Magnitude of derivatives affects the response behaviour of an airplane by altering the eigenvalues.
- Derivatives are function of the flying characteristics which change during the entire flight envelope.
- Control systems which provide artificial stability to an airplane having undesirable flying characteristics are commonly called as ***stability augmentation systems.***

SAS Design: Generic Philosophy

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Original system
$$\dot{X} = AX + BU$$
- Control Input
$$U = \underbrace{U_A}_{\text{Automatic}} + \underbrace{U_P}_{\text{Pilot input}} = -KX + U_P$$
- Modified system
$$\dot{X} = \left(A - BK\right)X + BU_P$$
$$= A_{CL}X + BU_P$$
- Philosophy: Design K such that A_{CL} has desired eigenvalues.

SAS Design for Stability Augmentation

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Problem: Determine the feedback gain K that produces the desired stability characteristics.
- SAS design:
  - Longitudinal stability augmentation design
  - Lateral stability augmentation design

Note: Handling quality improvement can be done using the same philosophy.

Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} X_\delta \\ Z_\delta \\ M_\delta \\ 0 \end{bmatrix}}_{B} \Delta\delta_e$$

The eigenvalues of stability matrix *A* are the short period and long period roots, which may be unacceptable to the pilot. If unacceptable, then let us design

$$\Delta\delta_e = -K\,X + \underbrace{\Delta\delta_e^P}_{\text{Pilot input}} = -\begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} X + \Delta\delta_e^P$$

Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Augmented CL system matrix

$$A_{CL} = \begin{bmatrix} X_u - X_\delta k_1 & X_w - X_\delta k_2 & -X_\delta k_3 & -g - X_\delta k_4 \\ Z_u - Z_\delta k_1 & Z_w - Z_\delta k_2 & u_0 - Z_\delta k_3 & -Z_\delta k_4 \\ M_u - M_\delta k_1 & M_w - M_\delta k_2 & M_q - M_\delta k_3 & -M_\delta k_4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Design the gain matrix design to place the eigenvalues at the desired locations following the "Pole placement philosophy"

Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- The characteristic equation for the augmented matrix is obtained by solving $|\lambda I - A_{CL}| = 0$, which yields a quartic characteristic equation

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

- Coefficients *A, B, C, D, E* are functions of known stability derivatives and the unknown feedback gains.

- Let the desired characteristic roots be

$$\lambda_1, \lambda_2 = -\zeta_{sp}\omega_{nsp} \pm \sqrt{1-\zeta_{sp}^2}, \quad \lambda_3, \lambda_4 = -\zeta_p\omega_{np} \pm \sqrt{1-\zeta_p^2}$$

$'sp'$: short period eigenvalues.

$'p'$: phugoid period eigenvalues

Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Desired characteristic equation

$$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)(\lambda - \lambda_4) = 0$$

$$i.e.$$

$$\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0$$

- Equate the coefficients and obtain the gain

$$\left.\begin{aligned} A &= 1 \\ B &= b \\ C &= c \\ D &= d \end{aligned}\right\} \text{Solve for } K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$

Example: Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Problem:

An airplane have poor short-period flying qualities in a particular flight regime. To improve the flying qualities, a stability augmentation system using state feedback is employed is to be employed. Determine the feedback gain so that the airplane's short-period characteristics are

$$\lambda_{sp} = -2.1 \pm 2.14\, i$$

Assume that the original short period dynamics is given by

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} -0.334 & 1 \\ -2.52 & -0.387 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} -0.027 \\ -2.6 \end{bmatrix} \Delta\delta_e$$

Example: Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Closed loop matrix $A_{CL} = \left(A - BK\right)$

$$= \begin{bmatrix} -0.334 + 0.027k_1 & 1 + 0.027k_2 \\ -2.52 + 2.6k_1 & -0.387 + 2.6k_2 \end{bmatrix}$$

Characteristic equation:

$$\begin{vmatrix} \lambda + 0.334 - 0.027k_1 & -1 - 0.027k_2 \\ 2.52 - 2.6k_1 & \lambda + 0.387 - 2.6k_2 \end{vmatrix} = 0$$

$$\lambda^2 + \left(0.721 - 0.027k_1 - 2.6k_2\right)\lambda + 2.65 - 2.61k_1 - 0.8k_2 = 0$$

Desired characteristic equation:

$$\lambda^2 + 4.2\lambda + 9 = 0$$

Example: Longitudinal SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Compare like powers of λ :

$0.721 - 0.027k_1 - 2.6k_2 = 4.2$

$2.65 - 2.61k_1 - 0.8k_2 = 9$

Solving for the gains yields:

$k_1 = -2.03$

$k_2 = -1.318$

The state feedback control is given by:

$$\Delta\delta_e = \left(2.03\Delta\alpha + 1.318\Delta q\right) + \underbrace{\Delta\delta_e^P}_{\text{Pilot input}}$$

Lateral SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

The linearized lateral state equations in state space form

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_u & 0 & -u_0 & g \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} Y_{\delta,a} & Y_{\delta,r} \\ L_{\delta,a} & L_{\delta,r} \\ N_{\delta,a} & N_{\delta,r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

A state feedback control law can be expressed as

$$U = \underbrace{U_A}_{\text{Automatic}} + \underbrace{U_P}_{\text{Pilot input}} = -CK\,X + U_P$$

The constant $C = [c_1 \; c_2]$ establishes the relationship between the aileron and rudder (control allocation).

$$\left(c_1 + c_2\right) = 1, \quad \left(c_1 / c_2\right) = \left(\Delta\delta_a / \Delta\delta_r\right)$$

Lateral SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Substituting the control equation into state equation yields

$$\dot{X} = (A - BCK)X + BU, \quad A_{CL} = (A - BCK)$$

The augmented characteristic equation is solved using the determinant $|\lambda I - \mathrm{A}|$

The desired characteristic equation is obtained through desired eigen values

$$\lambda_1 = \lambda_{directional} \quad \lambda_2 = \lambda_{spiral} \quad \lambda_3, \lambda_4 = -\zeta_{DR}\omega_{nDR} \pm \omega_{nDR}\sqrt{1 - \xi_{DR}^2}$$

Equate the coefficients of augmented and desired characteristic equations.

Sove the set of algebraic equations to get gain K.

Example: Lateral SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Lateral body rate dynamics:

$$\begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} L_p & L_r \\ N_p & N_r \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta r \end{bmatrix} + \begin{bmatrix} L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \end{bmatrix} \underbrace{\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}}_{U}$$

Control:

$$U = \underbrace{U_A}_{\text{Automatic}} + \underbrace{U_P}_{\text{Pilot input}} = -CK\,X + U_P$$

Closed loop system matrix:

$$A_{CL} = A - BCK = \begin{bmatrix} L_p - k_1\left(c_1 L_{\delta a} + c_2 L_{\delta r}\right) & L_r - k_2\left(c_1 L_{\delta a} + c_2 L_{\delta r}\right) \\ N_p - k_1\left(c_1 N_{\delta a} + c_2 N_{\delta r}\right) & N_r - k_2\left(c_1 N_{\delta a} + c_2 N_{\delta r}\right) \end{bmatrix}$$

Example: Lateral SAS

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Characteristic equation:

$$\left|\lambda I - A_{CL}\right| = 0$$

$$\lambda^2 + \left(k_1 L_c + k_2 N_c - L_p - N_r\right)\lambda + k_1\left(L_r N_c - N_r L_c\right)$$
$$+ k_2\left(N_p L_c - L_p N_c\right) + N_r L_p - N_p L_r = 0$$

where $L_c = c_1 L_{\delta a} + c_2 L_{\delta r},\ N_c = c_1 N_{\delta a} + c_2 N_{\delta r}$

Desired characteristic equation:

$$(\lambda - \lambda_1)(\lambda - \lambda_2) = \lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 = 0$$

By equating the like powers of λ

the feedback gains k_1 and k_2 can be obtained.

Cruise Control Systems

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Cruise Control Applications

- Attitude control (to maintain pitch, roll and heading)
- Altitude hold (to maintain a desired altitude)
- Speed control (to maintain constant speed or Mach no.)

Summary of LQR Design: Infinite Time Regulator Problem

Problem:

State equation: $\dot{X} = AX + BU$

Cost function: $J = \frac{1}{2}\int_0^{\infty}\left(X^T Q X + U^T R U\right) dt$

Solution:

Solve the ARE: $PA + A^T P - PBR^{-1}B^T P + Q = 0$

Compute the control: $U = -\left(R^{-1}B^T P\right)X = -KX$

Example: Roll stabilization system

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

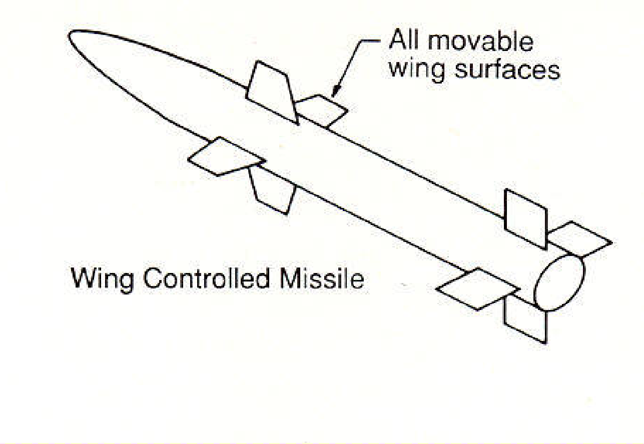
Guided missiles need roll orientation to be fixed for proper functioning of guidance unit. The objective here is to design a roll autopilot through feedback control.

System dynamics :

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & L_p \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ L_{\delta a} \end{bmatrix} \delta_a$$

where

$$L_p = \frac{1}{I_x}\left(\partial L \big/ \partial P \right), \quad L_{\delta a} = \frac{1}{I_x}\left(\partial L \big/ \partial \delta_a \right)$$



Example: Roll stabilization system

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

The quadratic performance index which needs to be minimized is

$$J = \frac{1}{2}\int_0^\infty \left[\left(\frac{\phi}{\phi_{\max}}\right)^2 + \left(\frac{p}{p_{\max}}\right)^2 + \left(\frac{\delta_a}{\delta_{a\max}}\right)^2 \right] dt$$

$\phi_{\max}$ = the maximum desired roll angle, $p_{\max}$ = the maximum roll rate

$\delta_{a\max}$ = the maximum aileron deflection

Comparing the PI with standard form gives Q and R as

$$Q = \begin{bmatrix} \frac{1}{\phi_{\max}^2} & 0 \\ 0 & \frac{1}{p_{\max}^2} \end{bmatrix}, \quad R = \frac{1}{\delta_{a\max}^2}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & L_p \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ L_{\delta a} \end{bmatrix}$$

Example: Roll stabilization system

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Algebraic Ricatti Equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

where $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$

Substituting matrices $A,\ B,\ Q$ and R into the Ricatti equation

$$\frac{1}{\phi_{\max}^2} - p_{12}^2 L_{\delta a}^2 \delta_{a\max}^2 = 0$$

$$p_{11} + p_{12} L_p - p_{12} p_{22} L_{\delta a}^2 \delta_{a\max}^2 = 0$$

$$2p_{12} + 2p_{22} L_p + \frac{1}{p_{\max}^2} - p_{22}^2 L_{\delta a}^2 \delta_{a\max}^2 = 0$$

Example: Roll stabilization system

Ref.: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Control Gain:

$$K = R^{-1}B^T P = \delta_{a\max}^2 \begin{bmatrix} 0 & L_{\delta_a} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$$

Optimal controller:

$$\delta_a = -K\,X = -\delta_{a\max}^2 \begin{bmatrix} L_{\delta a} p_{12} & L_{\delta a} p_{22} \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix}$$

Note : MATLAB function for solving the LQR problems are 'lqr' and 'lqr2'.

Summary

Applications of Automatic Flight Control Systems can lead to:

- Stability Augmentation Systems
- Cruise Control Systems
- Landing Aids
- **Automatic path planning and guidance**

Both classical as well as modern control techniques can be utilized for the above purpose. However, modern control techniques can deal with MIMO plants more naturally and effectively.

Thanks for the Attention….!