Lecture – 29

Applications Linear Control Design Techniques in Aircraft Control – I

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Topics

- Brief Review of Aircraft Flight Dynamics
- An Overview of Automatic Flight Control Systems
- Automatic Flight Control Systems: Classical (Frequency Domain) Designs

Brief Review of Aircraft Flight Dynamics

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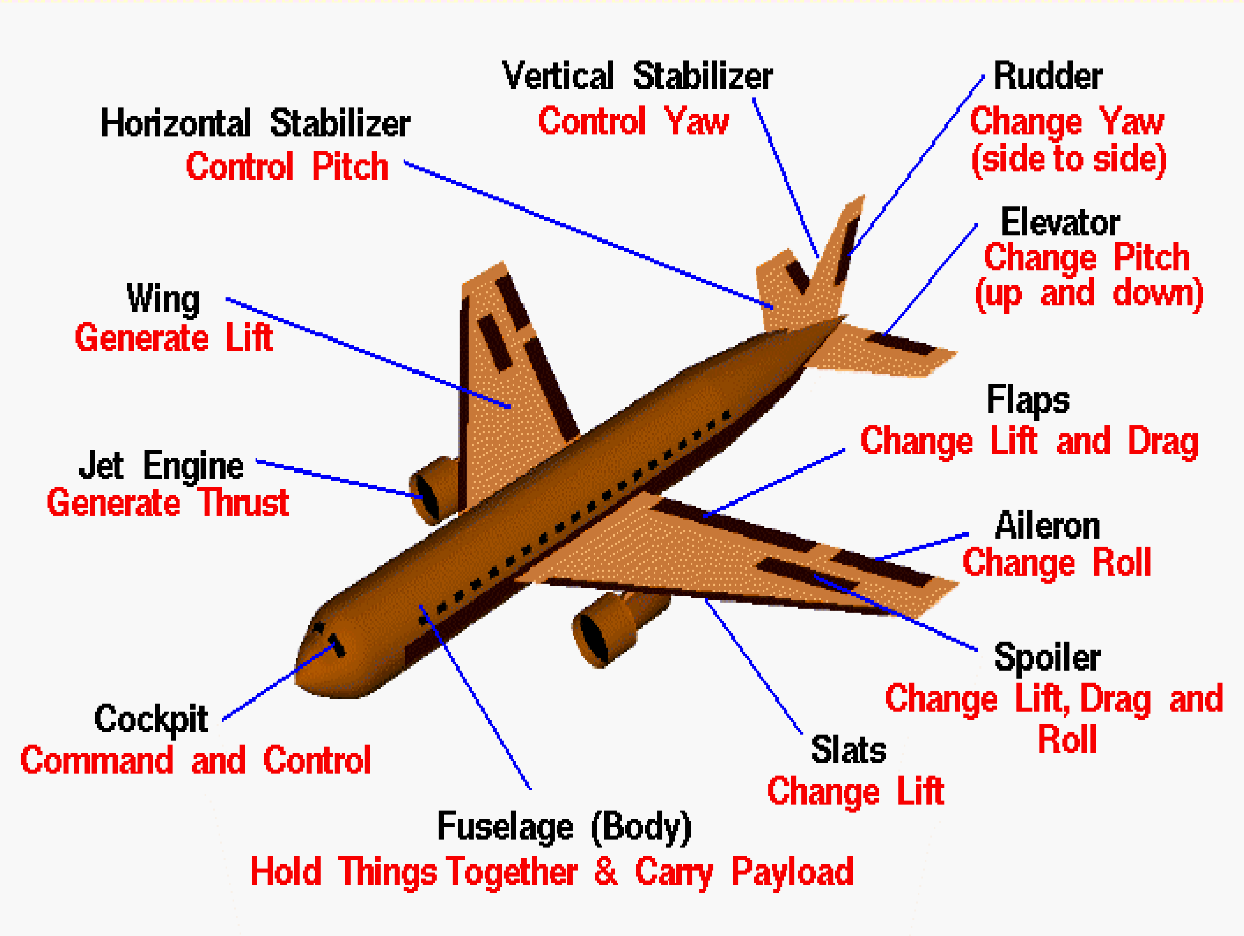
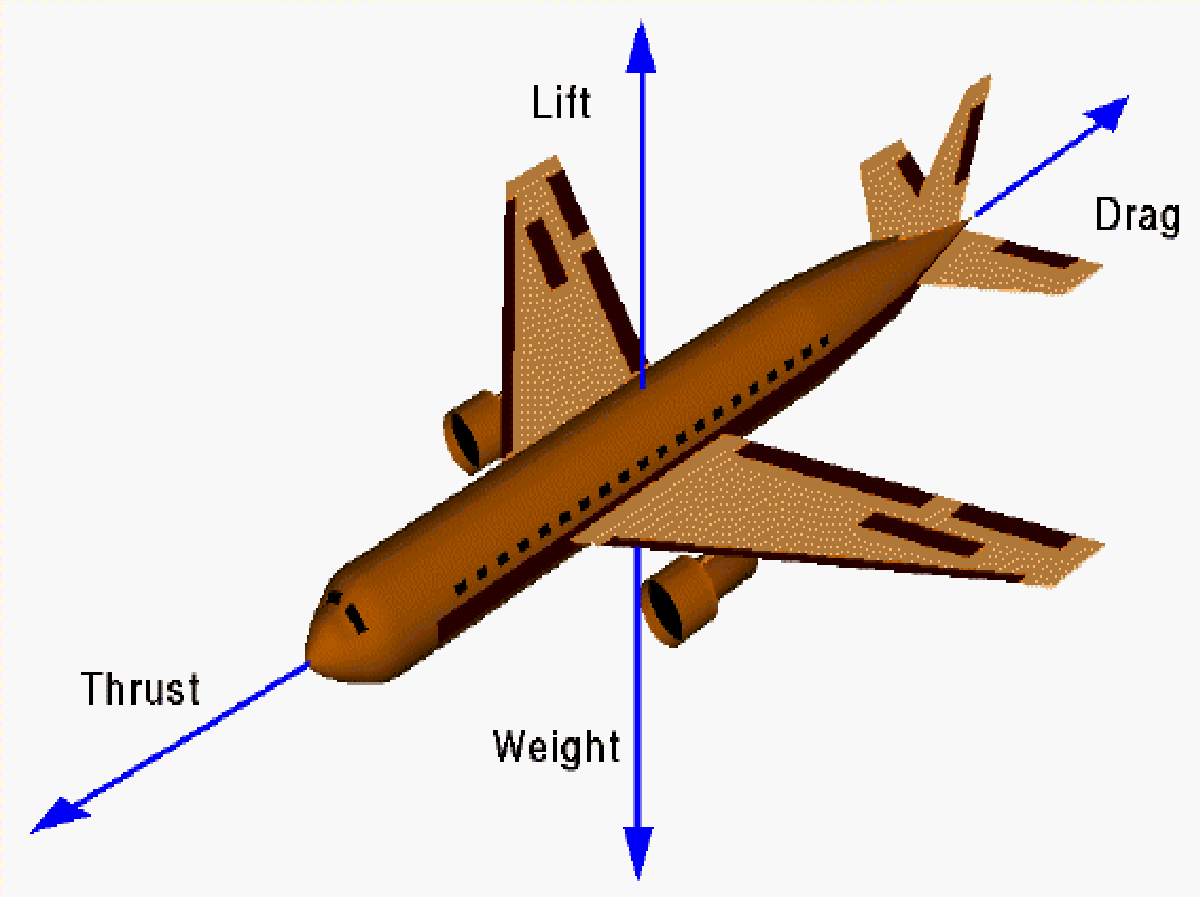
Geometry of Conventional Aircrafts



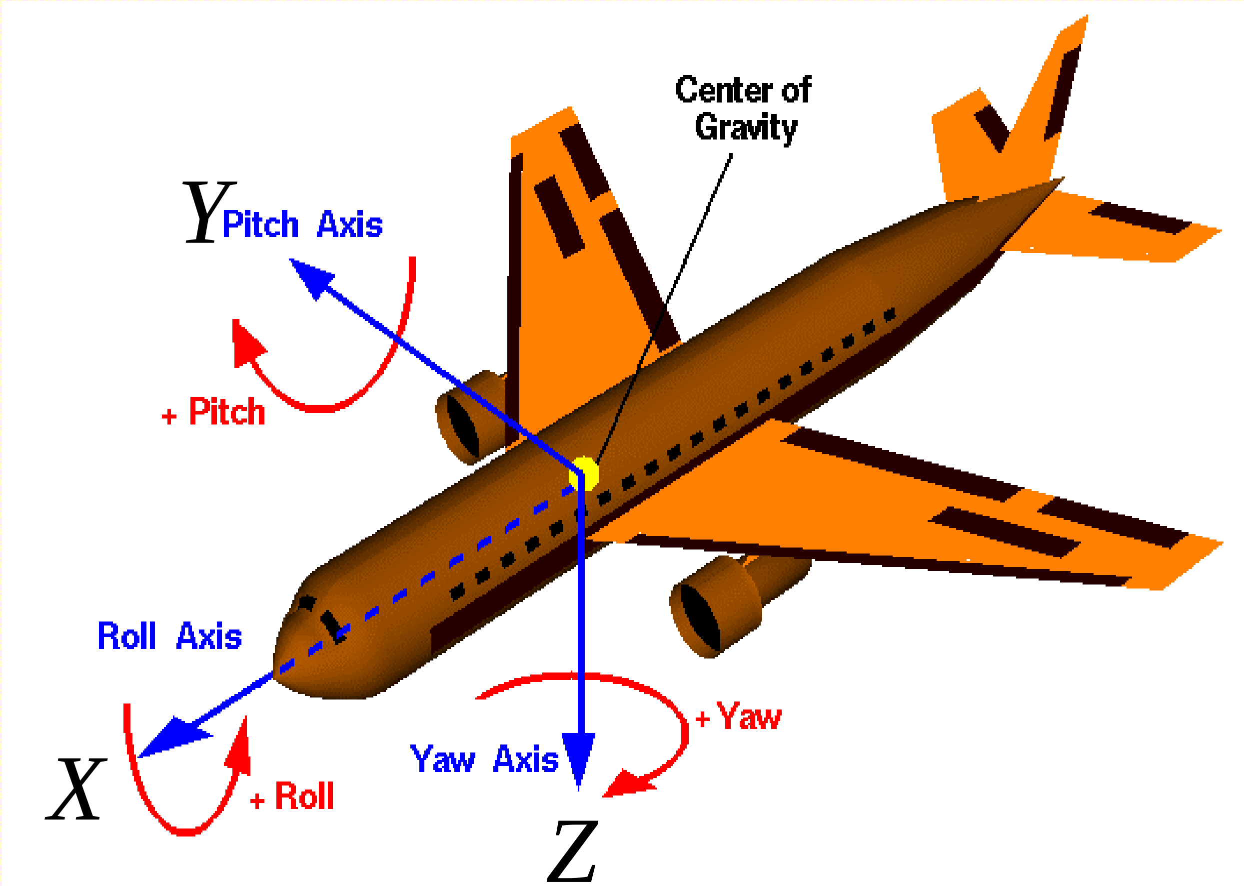
Basic Force Balance

- Weight
- Lift
- Drag
- Thrust

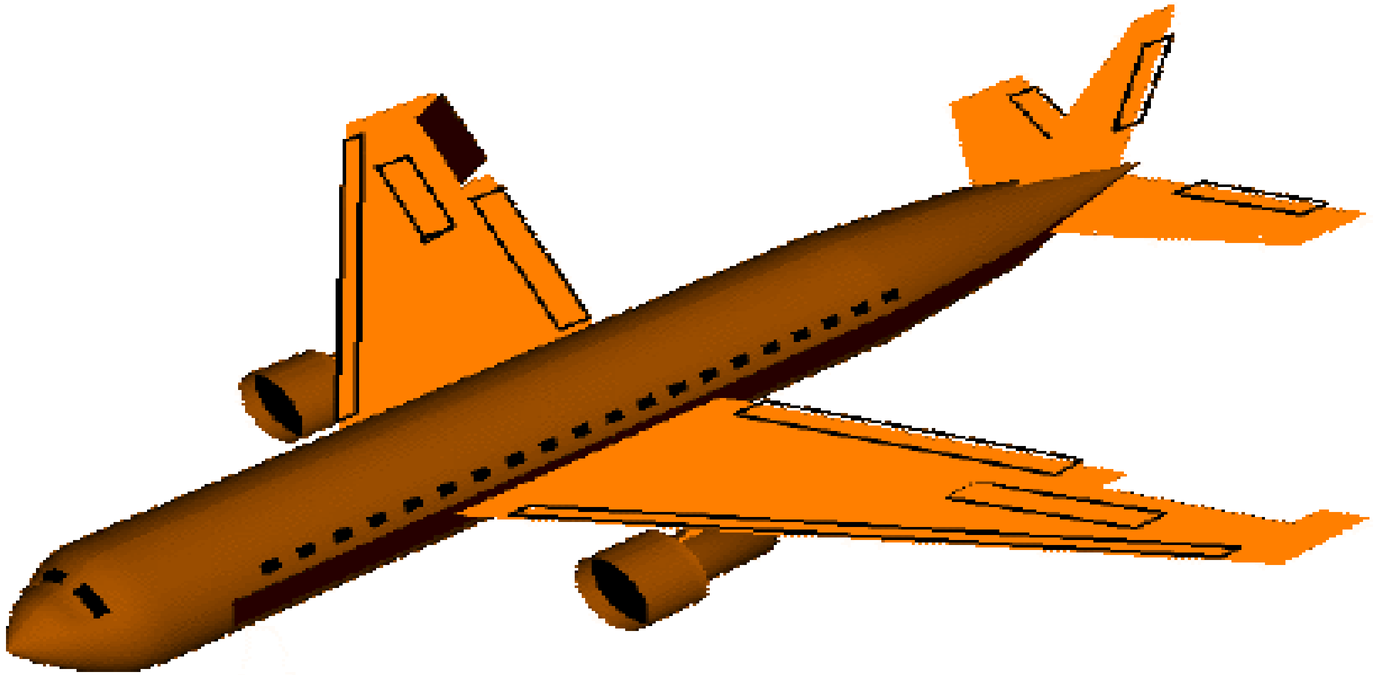


Basic Moment Balance

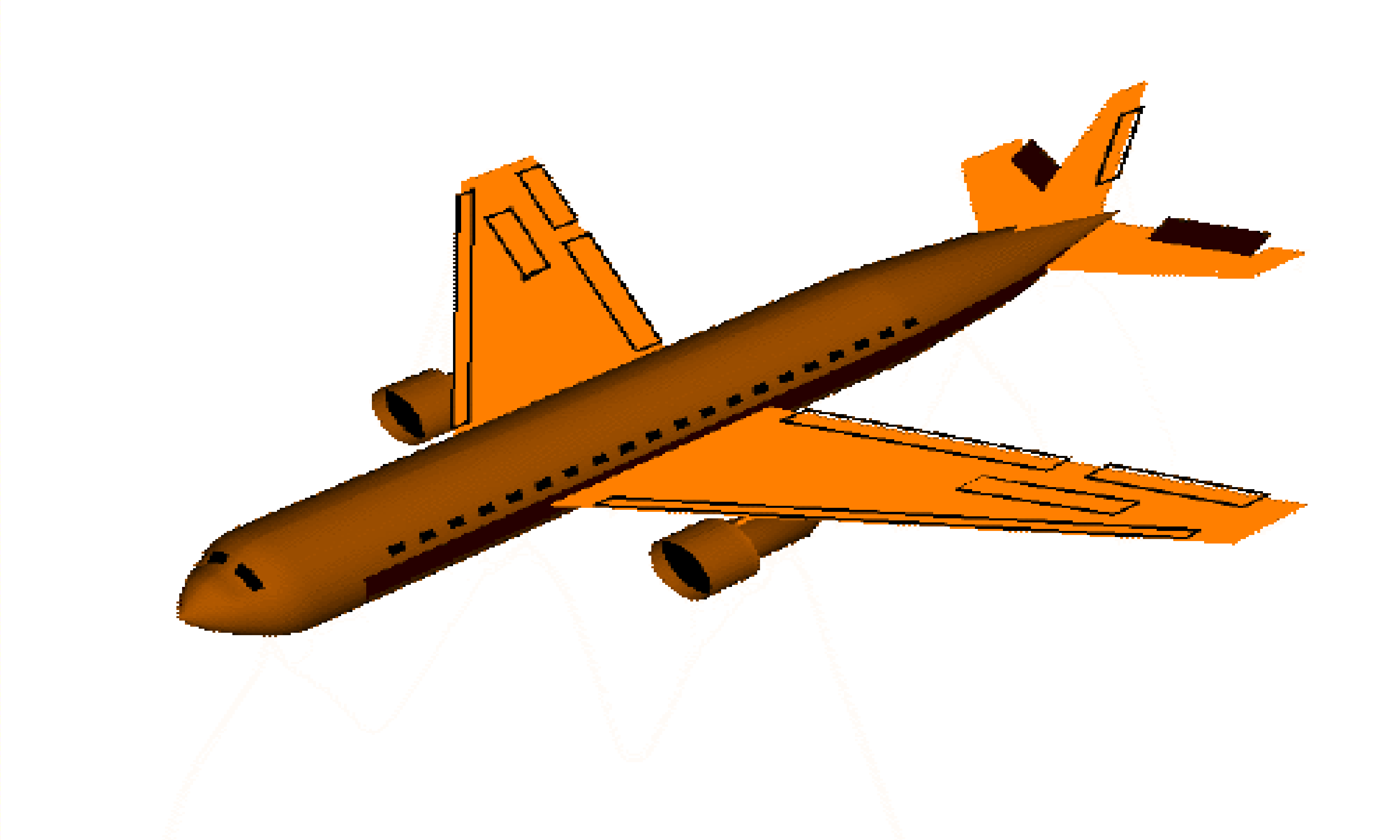
- Rolling
- Pitching
- Yawing



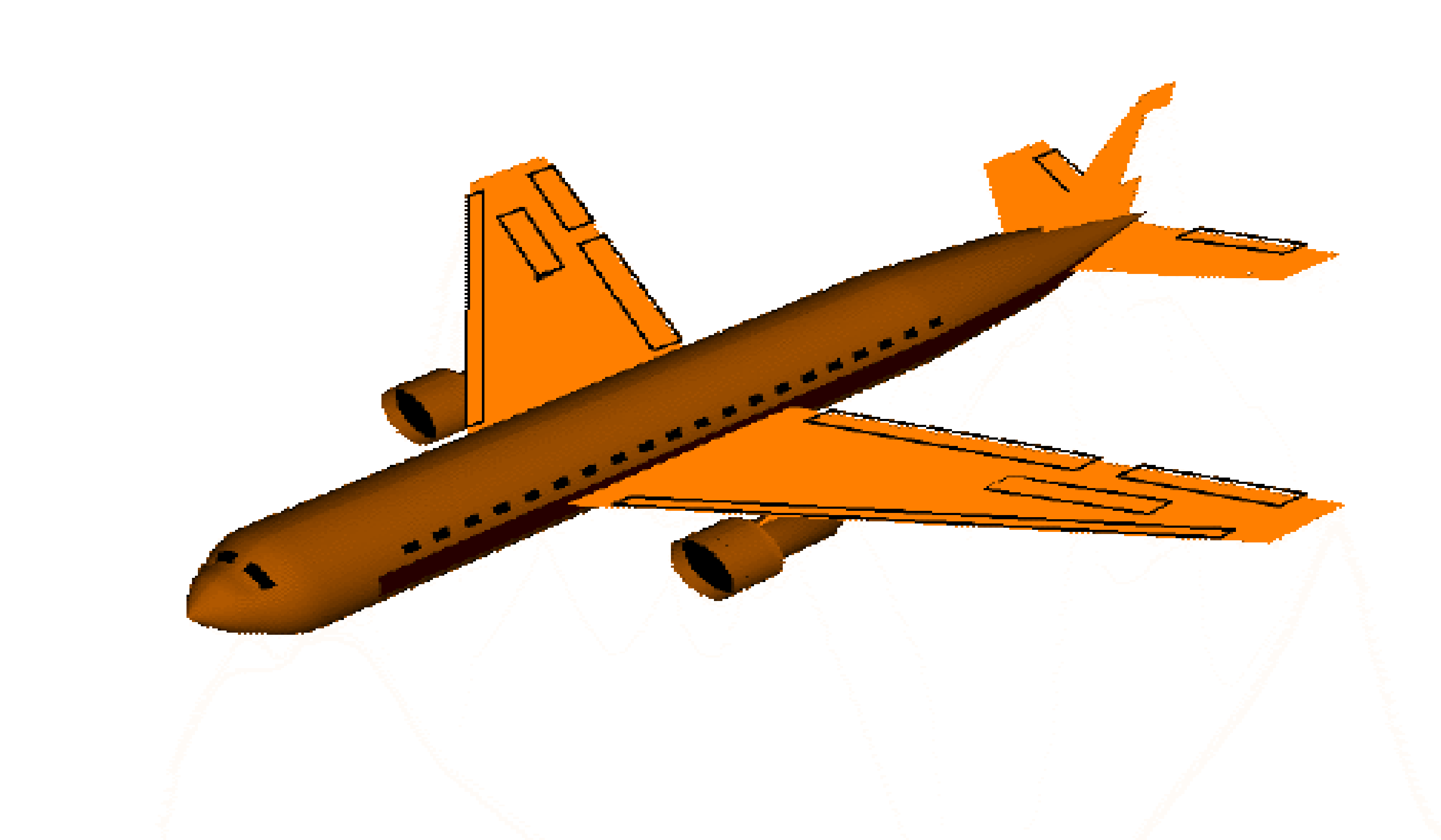
Aileron ⇨ Roll



Elevator ⇨ Pitch



Rudder ⇨ Yaw



Airplane Dynamics: Six Degree-of-Freedom Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, *1995*

$$\dot{U} = VR - WQ - g\sin\Theta + \frac{1}{m}(X + X_T)$$

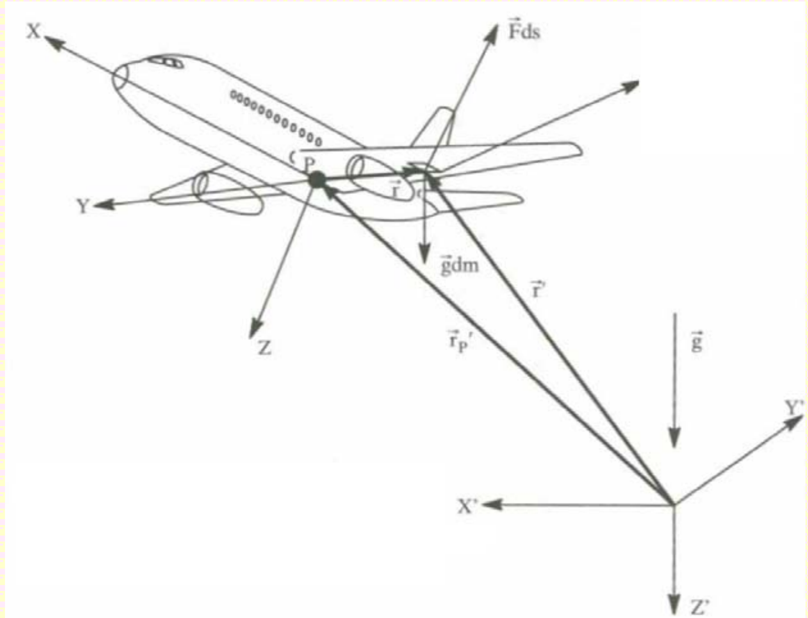
$$\dot{V} = WP - UR + g\sin\Phi\cos\Theta + \frac{1}{m}(Y + Y_T)$$

$$\dot{W} = UQ - VP + g\cos\Phi\cos\Theta + \frac{1}{m}(Z + Z_T)$$

$$\dot{P} = c_1 QR + c_2 PQ + c_3 (L + L_T) + c_4 (N + N_T)$$

$$\dot{Q} = c_5 PR - c_6 (P^2 - R^2) + c_7 (M + M_T)$$

$$\dot{R} = c_8 PQ - c_2 QR + c_4 (L + L_T) + c_9 (N + N_T)$$



$$\dot{\Phi} = P + Q\sin\Phi\tan\Theta + R\cos\Phi\tan\Theta$$

$$\dot{\Theta} = Q\cos\Phi - R\sin\Phi$$

$$\dot{\Psi} = (Q\sin\Phi + R\cos\Phi)\sec\Theta$$

$$\begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{z}_I \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$\dot{h} = -\dot{z}_I = U\sin\Theta - V\cos\Theta\sin\Phi - W\cos\Theta\cos\Phi$$

Representation of Longitudinal Dynamics in Small Perturbation

State space form:

$$\dot{X} = AX + BU_c$$

$$A = \begin{bmatrix} X_U & X_W & 0 & -g \\ Z_U & Z_W & U_0 & 0 \\ M_U + M_{\dot{W}} Z_U & M_W + M_{\dot{W}} Z_W & M_Q + M_{\dot{W}} U_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad X = \begin{bmatrix} \Delta U \\ \Delta W \\ \Delta Q \\ \Delta \theta \end{bmatrix}$$

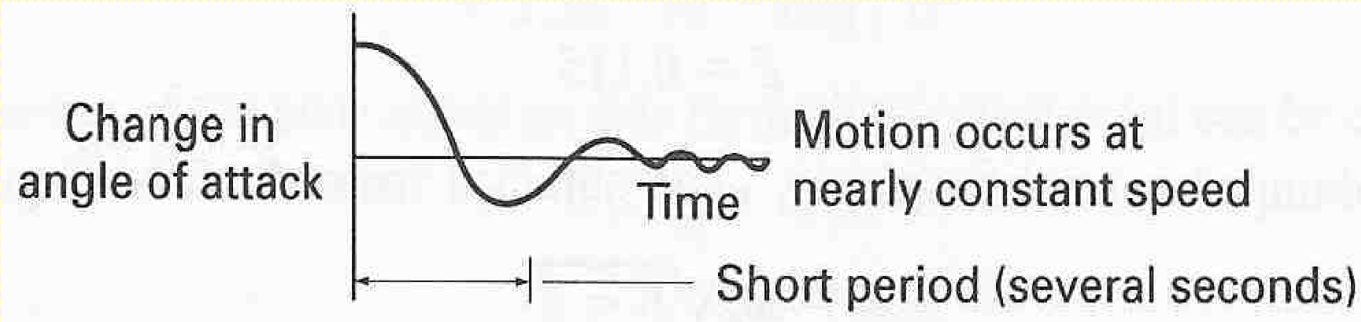
$$B = \begin{bmatrix} X_{\delta_E} & X_{\delta_T} \\ Z_{\delta_E} & Z_{\delta_T} \\ M_{\delta_E} + M_{\dot{W}} Z_{\delta_E} & M_{\delta T} + M_{\dot{W}} Z_{\delta_T} \\ 0 & 0 \end{bmatrix} \qquad U_c = \begin{bmatrix} \Delta \delta_E \\ \Delta \delta_T \end{bmatrix}$$

$$X_U = \frac{1}{m}\left(\frac{\partial X}{\partial U}\right), \quad X_W = \frac{1}{m}\left(\frac{\partial X}{\partial W}\right) \quad etc.$$

Short Period Mode

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Heavily damped
- Short time period
- Constant velocity



Short Period Dynamics

State Space Equation: $$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} a_{11} & 1 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Delta\delta_E$$

Transfer Function Equations:

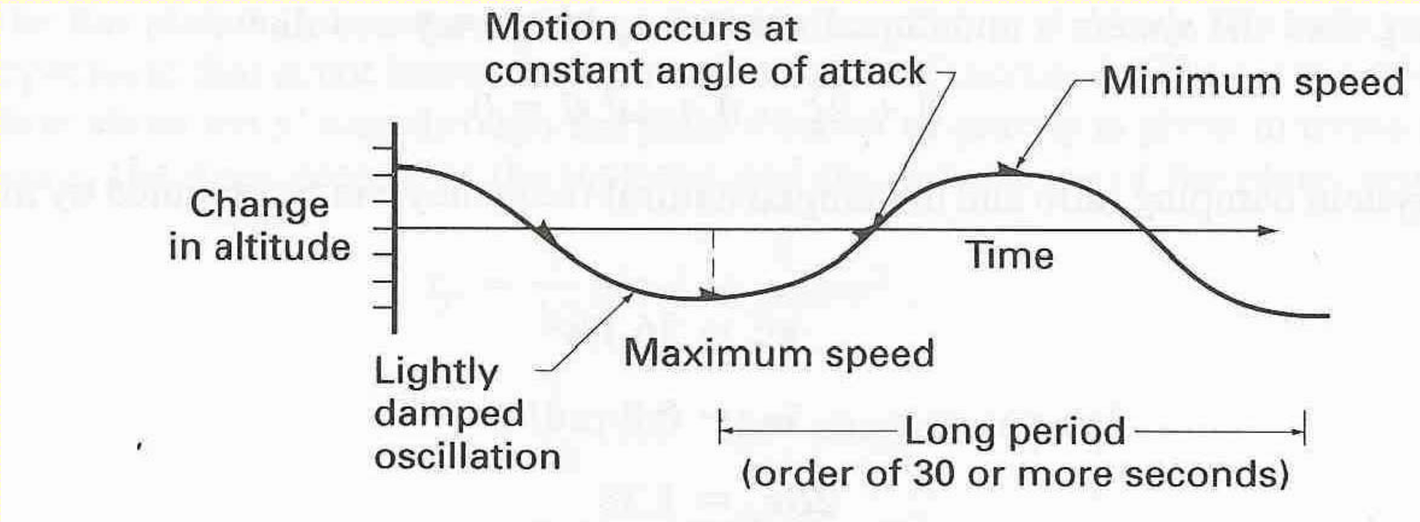
$$\frac{\Delta\alpha(s)}{\Delta\delta_E(s)} = \frac{A_\alpha s + B_\alpha}{As^2 + Bs + C}$$

$$\frac{\Delta q(s)}{\Delta\delta_E(s)} = \frac{A_q s + B_q}{As^2 + Bs + C}$$

Long Period (Phugoid) Dynamics

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Lightly damped
- Changes in pitch attitude, altitude, velocity
- Constant angle of attack



Long Period (Phugoid) Dynamics

State Space Equation:

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} a_{11} & -g \\ a_{21} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_E \\ \Delta \delta_T \end{bmatrix}$$

Transfer Function Equations:

Assumption: $\Delta \delta_T = 0$

$$\frac{\Delta u(s)}{\Delta \delta_E(s)} = \frac{A_u s + B_u}{A s^2 + B s + C}$$

$$\frac{\Delta \theta(s)}{\Delta \delta_E(s)} = \frac{A_\theta s + B_\theta}{A s^2 + B s + C}$$

Representation of Lateral Dynamics in Small Perturbation

State space form: $\dot{X} = AX + BU_c$

$$A = \begin{bmatrix} Y_V & Y_P & -(U_0 - Y_R) & g\cos\theta_0 \\ L_V^* + \dfrac{I_{XZ}}{I_X} N_V^* & L_P^* + \dfrac{I_{XZ}}{I_X} N_P^* & L_R^* + \dfrac{I_{XZ}}{I_X} N_R^* & 0 \\ N_V^* + \dfrac{I_{XZ}}{I_Z} L_V^* & N_P^* + \dfrac{I_{XZ}}{I_Z} L_P^* & N_R^* + \dfrac{I_{XZ}}{I_Z} L_R^* & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} \Delta V \\ \Delta P \\ \Delta R \\ \Delta \phi \end{bmatrix}$$

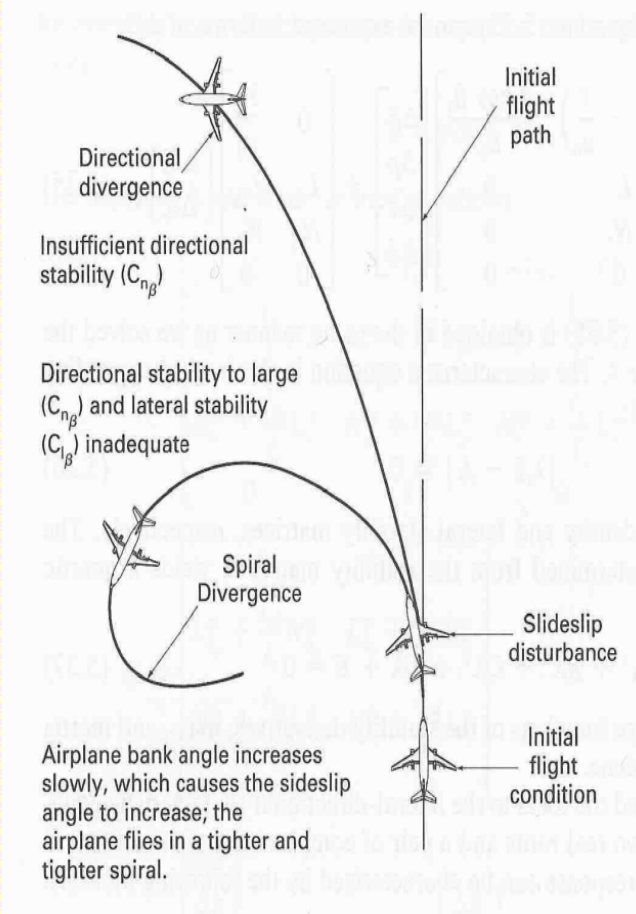
$$B = \begin{bmatrix} 0 & Y_{\delta_R} \\ L_{\delta_A}^* + \dfrac{I_{XZ}}{I_X} N_{\delta_A} & L_{\delta_R}^* + \dfrac{I_{XZ}}{I_X} N_{\delta_R} \\ N_{\delta_A}^* + \dfrac{I_{XZ}}{I_Z} L_{\delta_A}^* & N_{\delta_R}^* + \dfrac{I_{XZ}}{I_Z} L_{\delta_R}^* \\ 0 & 0 \end{bmatrix} \qquad U_c = \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_R \end{bmatrix}$$

Lateral Dynamic Instabilities

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Directional divergence

- Do not possess directional stability
- Tend towards ever-increasing angle of sideslip
- Largely controlled by rudder



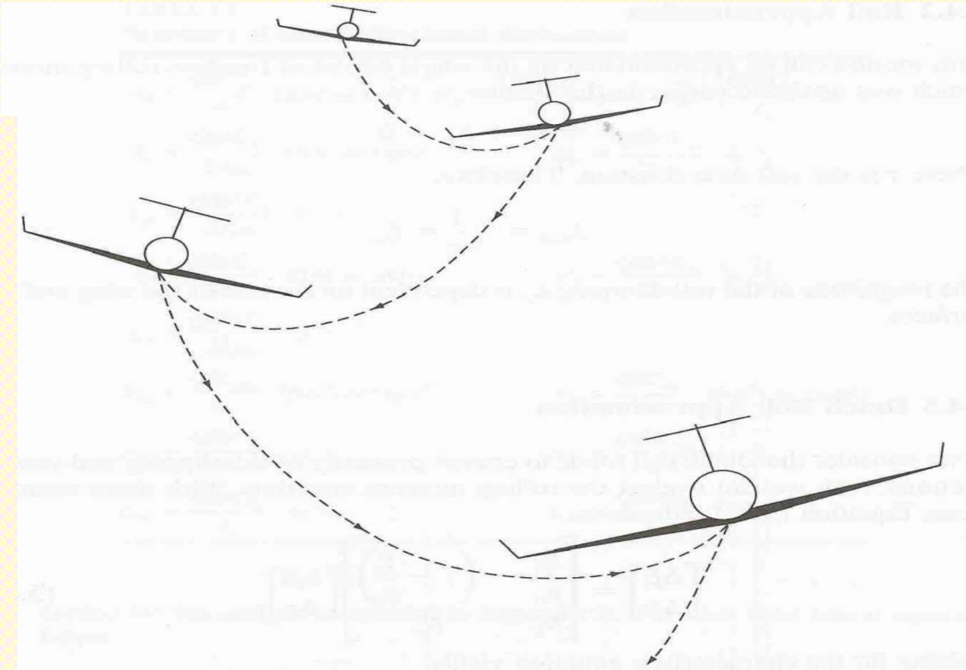
Spiral divergence

- Spiral divergence tends to gradual spiraling motion & leads to high speed spiral dive
- Non–oscillatory divergent motion
- Largely controlled by ailerons

Lateral Dynamic Instabilities

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Dutch roll oscillation



- Coupled directional-spiral oscillation
- Combination of rolling and yawing oscillation of same frequency but out of phase each other
- Controlled by using both ailerons and rudders

Dutch Roll Dynamics

State Space Equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_R \end{bmatrix}$$

Transfer Function Equations:

$$\frac{\Delta\beta(s)}{\Delta\delta_A(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C} \qquad \frac{\Delta\beta(s)}{\Delta\delta_R(s)} = \frac{\hat{A}_\beta s + \hat{B}_\beta}{As^2 + Bs + C}$$

$$\frac{\Delta r(s)}{\Delta\delta_A(s)} = \frac{A_r s + B_r}{As^2 + Bs + C} \qquad \frac{\Delta r(s)}{\Delta\delta_R(s)} = \frac{\hat{A}_r s + \hat{B}_r}{As^2 + Bs + C}$$

Automatic Flight Control Systems: An Overview

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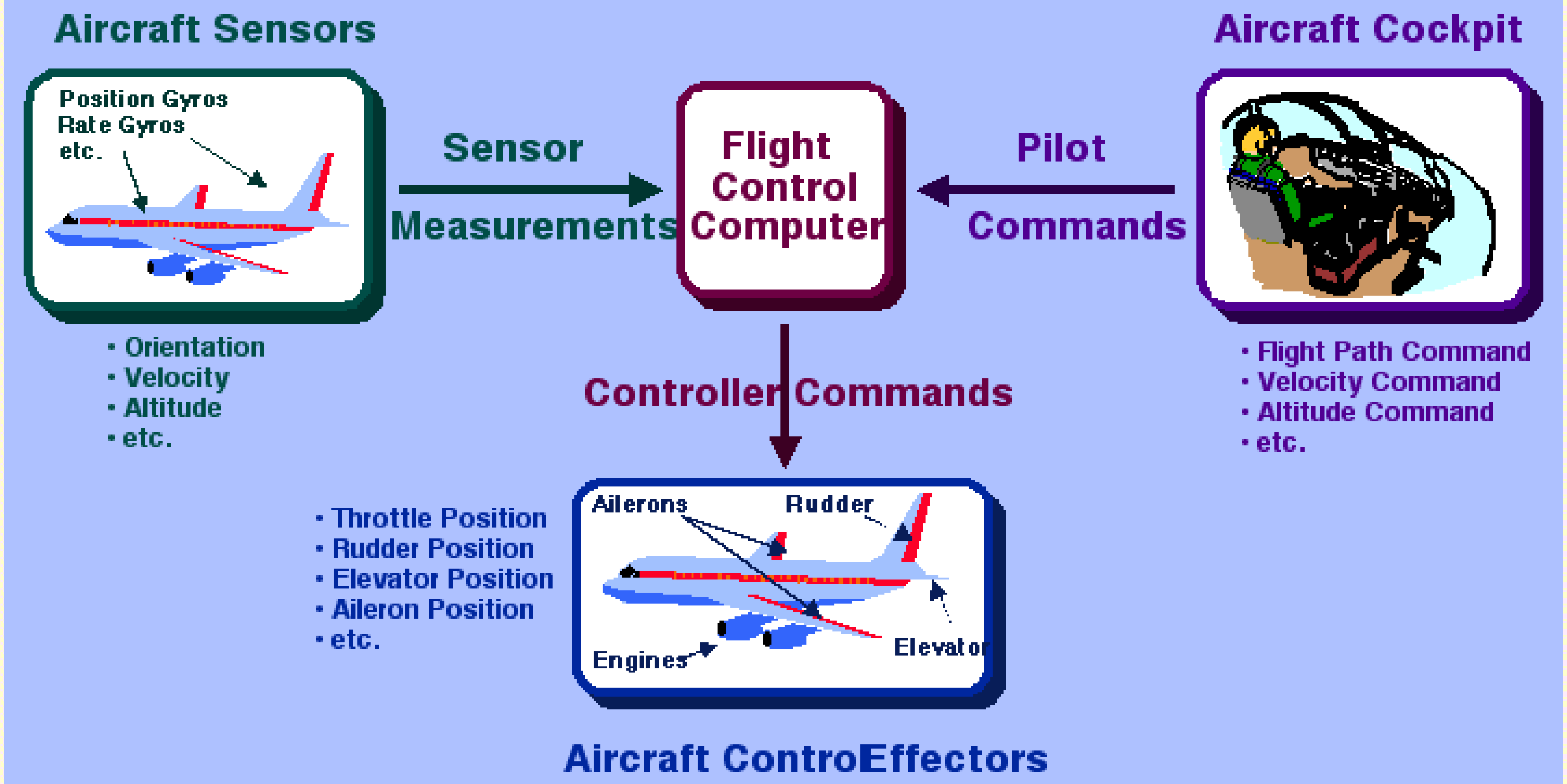
“Putting it All Together”

Flight Control System



Sensors

- **Altimeter:** Height above sea level
- **Air Data System:** Airspeed, Angle of Attack, Mach No., Air Temperature etc.
- **Magnetometer:** Heading
- **Inertial Navigation System (INS)**
  - **Accelerometers:** Translational motion of the aircraft in the three axes
  - **Gyroscopes:** Rotational motion of the aircraft in the three axes
- **GPS:** Accurate position, ground speed

The transfer function for most sensors can be approximated by a gain k

Actuators

- Electrical actuators:
  - Its a second order system in general
  - It can be approximated to a first order system with small angle displacements
- Hydraulic / Pneumatic actuators
  - First order system
- Combination of the above

Applications of Automatic Flight Control Systems

- Cruise Control Systems
  - Attitude control (to maintain pitch, roll and heading)
  - Altitude hold (to maintain a desired altitude)
  - Speed control (to maintain constant speed or Mach no.)
- Stability Augmentation Systems
  - Stability enhancement
  - Handling quality enhancement
- Landing Aids
  - Alignment control (to align wrt. runway centre line)
  - Glideslope control
  - Flare control

Techniques for Autopilot Design

- Frequency domain techniques:
  - Root locus
  - Bode plot
  - Nyquist plot
  - PID design etc.
- Time domain techniques:
  - Pole placement design
  - Lyapunov design
  - Optimal control design etc.

Automatic Flight Control Systems: Frequency Domain Designs

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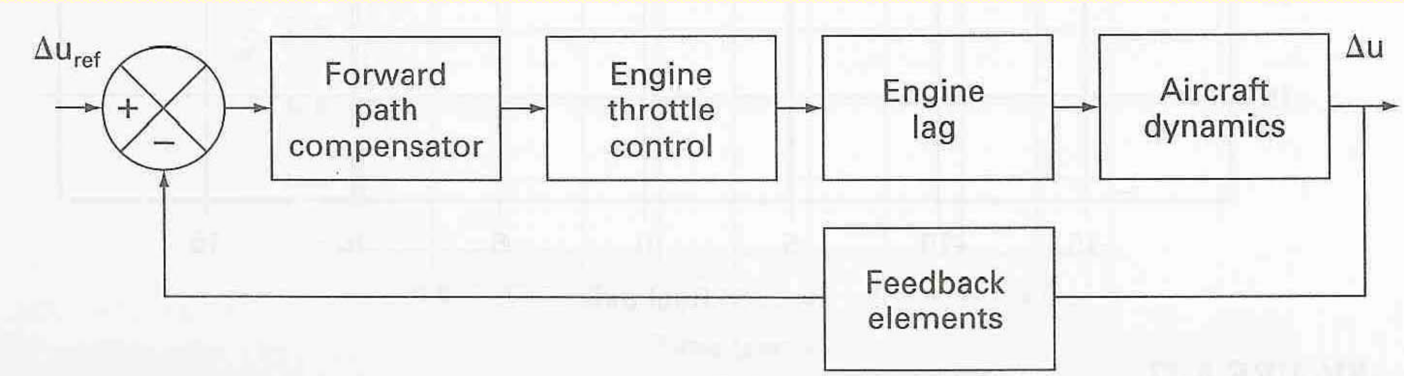
Velocity Hold Control System

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

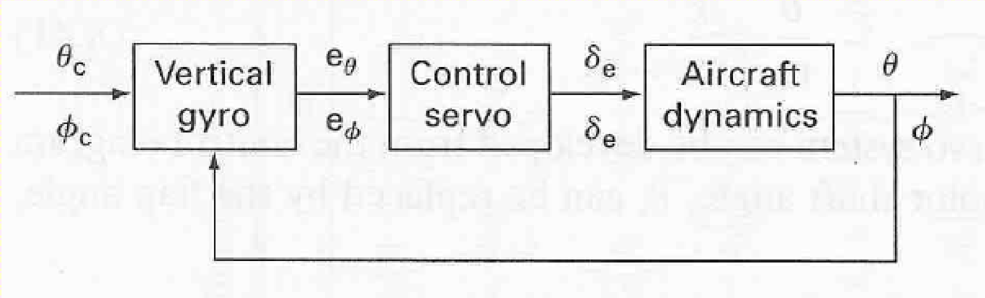
- The forward speed of an airplane can be controlled by changing the thrust produced by propulsion system.
- The function of the speed control system is to maintain the some desired flight speed.

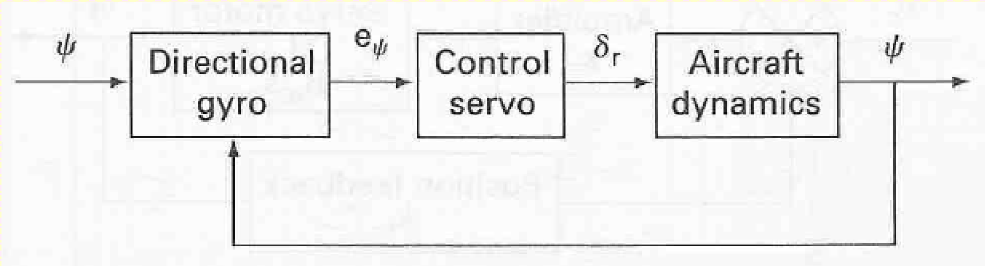


Attitude Control System

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

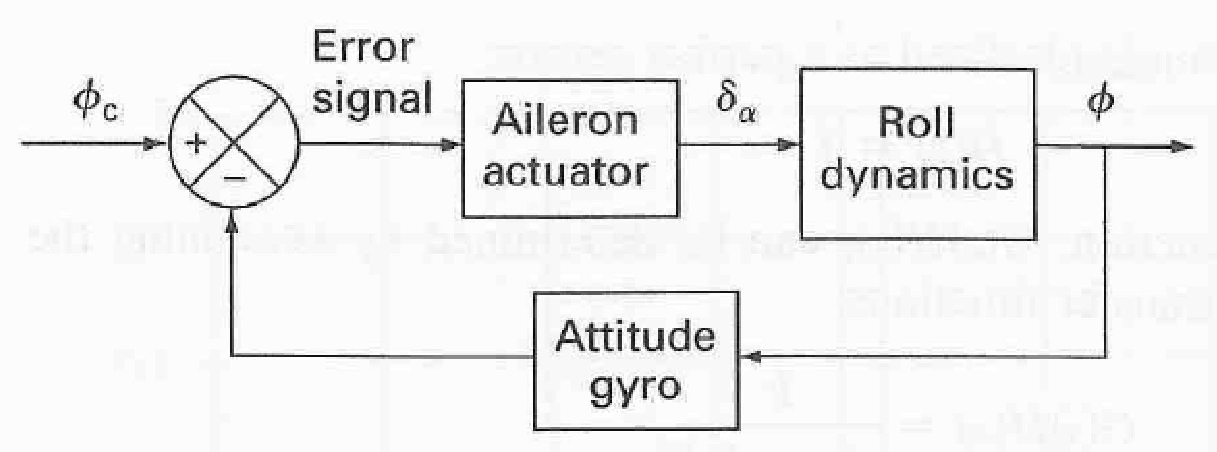
- Sense the angular position
- Compare the angular position with the desired angular position
- Generate commands proportional to the error signal





Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.



Components:

- Model for roll dynamics (transfer function)
- Attitude gyro
- Comparator
- Aileron actuator

Reference:
R. C. Nelson: *Flight Stability and Automatic Control*, McGraw-Hill, 1989.

Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Problem:

Design a roll attitude control system to maintain a wings level attitude for a vehicle having the following characteristics.

$$L_p = -0.5\, rad/s \quad L_{\delta a} = 2.0/s^2$$

The system performance is to have damping ratio $\zeta = 0.707$ and an undamped natural frequency $\omega_n = 10$ rad/s.

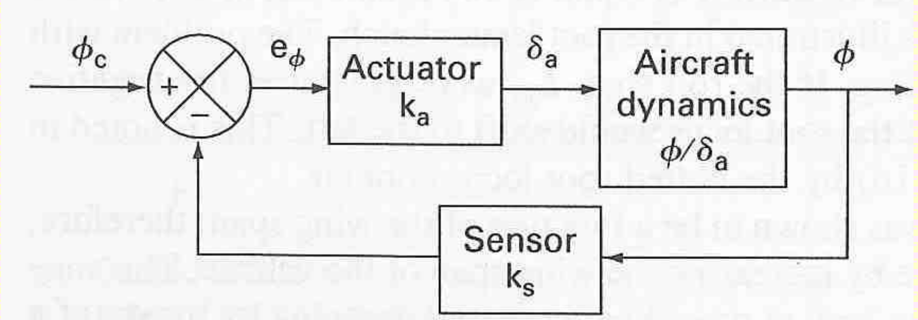
Assume the aileron actuator and the sensor (gyro) can be represented by the gains k_a and k_s

Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

The system transfer function

$$\frac{\Delta\phi(s)}{\Delta\delta_a(s)} = \frac{L_{\delta a}}{s(s - L_p)}$$



Forward path transfer function

$$G(s) = \frac{\Delta\delta_a(s)}{e(s)} \frac{\Delta\phi(s)}{\Delta\delta_a(s)} = k_a \frac{L_{\delta a}}{s(s - L_p)}$$

$$H(s) = k_s = 1 \quad \text{(unity feedback assumption)}$$

The loop transfer function

$$G(s)H(s) = \frac{k}{s(s + 0.5)}, \quad \text{where } k = k_a L_{\delta a}$$

Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

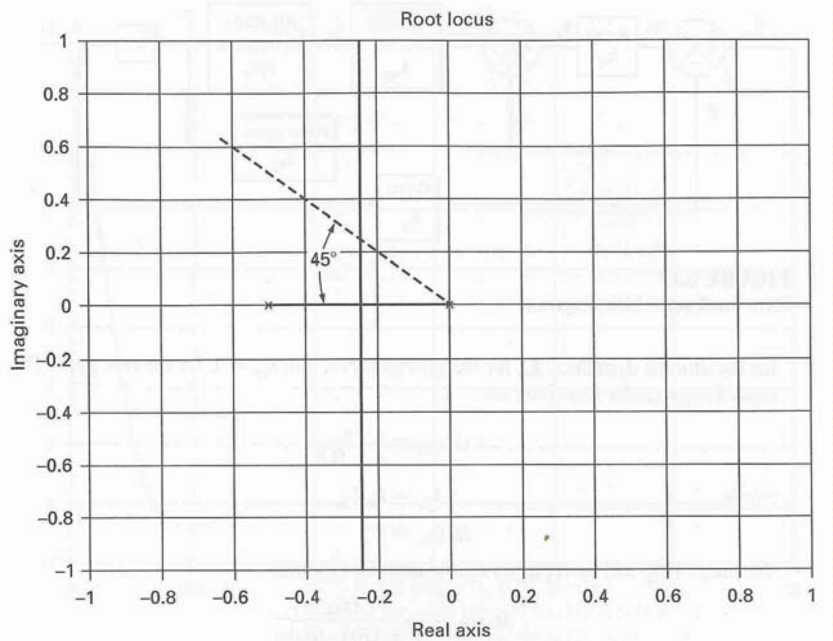
The desired damping needed is $\zeta = 0.707$

We know $\zeta = \cos\theta$. Hence,

draw a line of 45^0 from the origin.

Any root intersecting this line

will have $\zeta = 0.707$.



Gain is determined from:

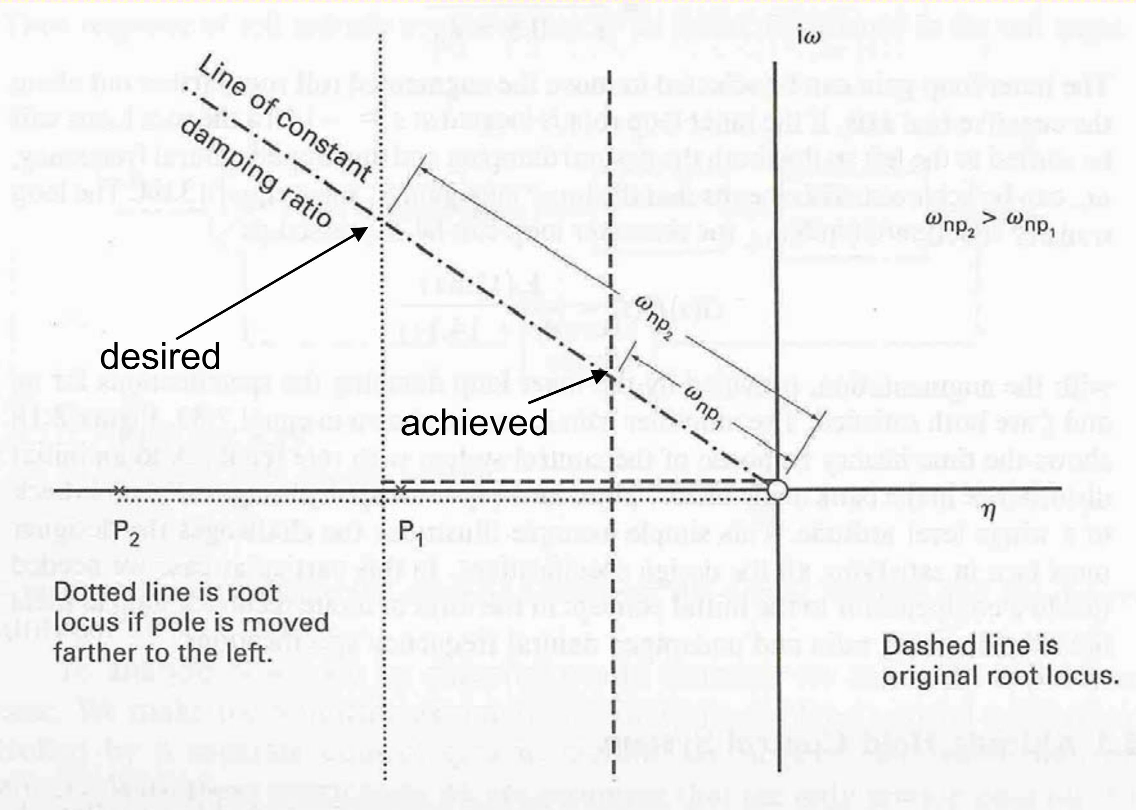
$$\frac{|k|}{|s||s+0.5|} = 1,$$

This leads to $k = 0.0139$. However,

$\omega_n = 0.35\, rad/s$ (much lower than desired!)

Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.



Moving pole P_1 to P_2 is impractical, unless there is an increase in wingspan of the aircraft..!

Hence, the cure is to have a stability augmentation system.

Roll Attitude Autopilot

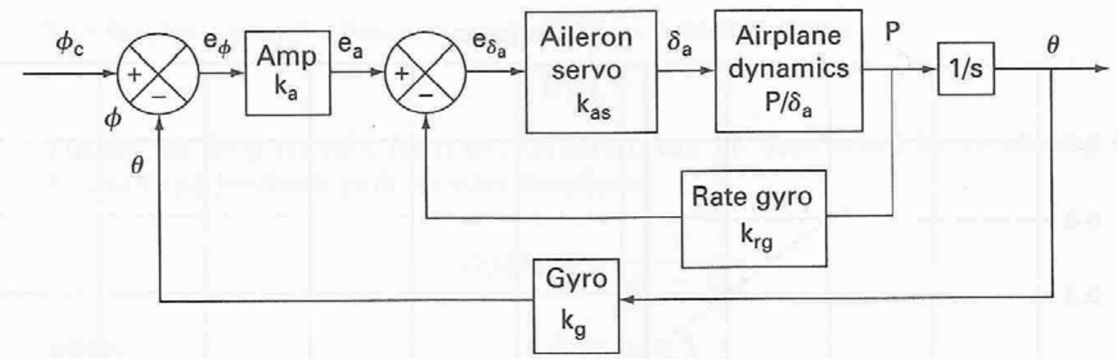
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Compensator is added in form of rate feedback loop to meet desired damping and natural frequency.
- The inner loop transfer function can be expressed as follows:

$$\frac{\Delta p(s)}{\Delta \delta_a(s)} = \frac{L_{\delta a}}{(s - L_p)}$$

- Transfer function of inner loop

$$G(s)_{IL} = \frac{k_{IL}}{s+0.5}, \;\; H(s)_{IL} = 1\,(k_{rg}), \;\; k_{IL} = k_{as} L_{\delta a}$$



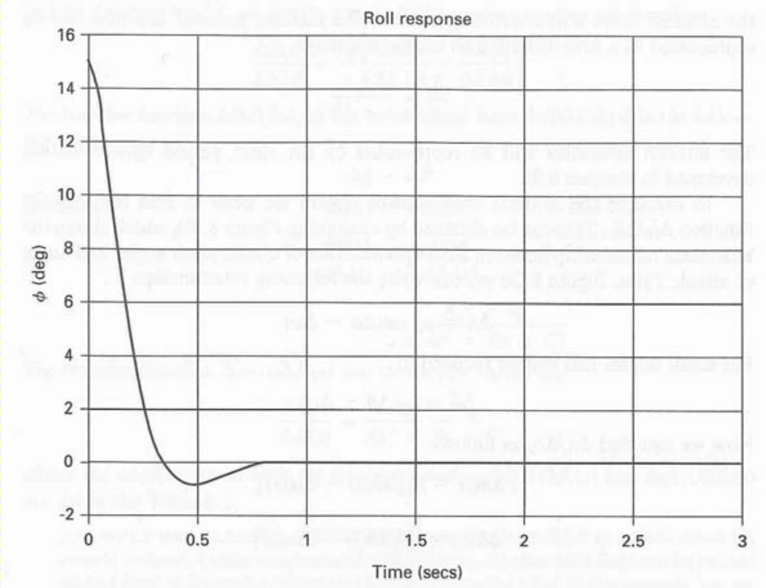
$$\mathrm{M(s)_{IL}} = \frac{G(s)_{IL}}{1 + G(s)_{IL} H(s)_{IL}} = \frac{k_{IL}}{s + 0.5 + k_{IL}}$$

Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Inner loop gain is selected to move the augmented root farther on the negative real axis.

If the inner loop is located at $s = -14.14,$ then the root locus will shift to the left and desired ξ and ω_n can be achieved.



Stability Augmentation System (SAS)

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Inherent stability of an airplane depends on the aerodynamic stability derivatives.
- Magnitude of derivatives affects both damping and frequency of the longitudinal and lateral motion of an airplane.
- Derivatives are function of the flying characteristics which change during the entire flight envelope.
- Control systems which provide artificial stability to an airplane having undesirable flying characteristics are commonly called as ***stability augmentation systems.***

Example: SAS

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Consider an aircraft with poor short period dynamic characteristics. Assume one degree of freedom (only pitching motion about CG) to demonstrate the SAS.

Short period dynamics:

$$\ddot{\theta} - \left(M_q + M_\alpha\right)\dot{\theta} + M_\alpha \theta = M_\delta \delta$$

Substituting the numerical values

$$\ddot{\theta} - 0.071\dot{\theta} + 5.49\theta = -6.71\delta_e$$

This leads to $\zeta_{sp} = 0.015 \quad \omega_{nsp} = 2.34\, rad/s$

It is seen that the airplane has poor damping (flying quality).

Example: SAS

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

One way to improve damping is to provide rate feedback.

Artificial damping is provided by producing an elevator deflection in proportion to pitch rate and then adding it to the pilot's input; i.e.

$$\delta_e = \underbrace{\delta_{ep}}_{\substack{Pilot\\input}} + \underbrace{k\dot{\theta}}_{\substack{Artificial\\command}}$$

Rate gyro measures the $\dot{\theta}$ and creates an electrical signal to provide $k\dot{\theta}$ in addtion to δ_e.

With this, the modified dynamics becomes:

$$\ddot{\theta} - \left(0.071 + 6.71k\right)\dot{\theta} + 5.49\theta = -6.71\delta_e$$

$$2\zeta\omega_n = 0.071 + 6.71k, \quad \omega_n^2 = 5.49$$

Hence, by varying k, the desired damping is achieved.

Landing System

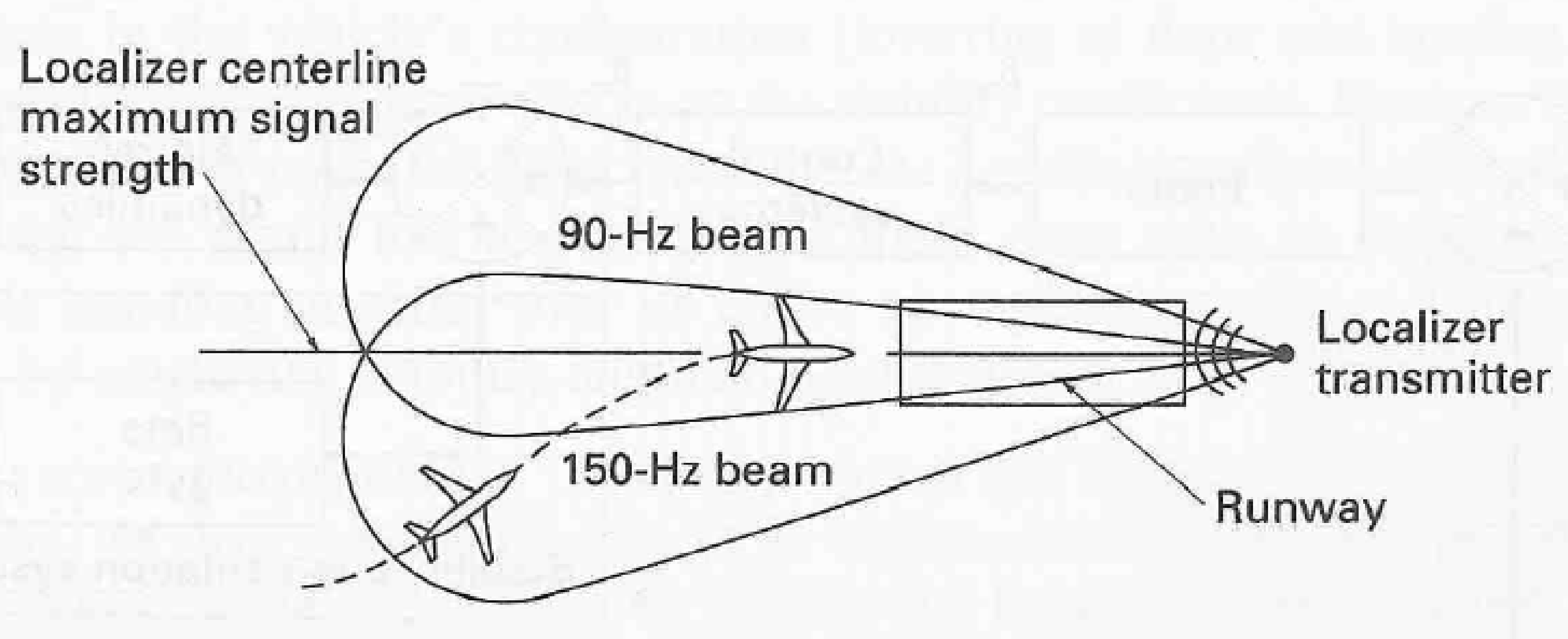
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Key Components:

- Alignment control (to align wrt. runway centre line)
- Glideslope control
- Flare control

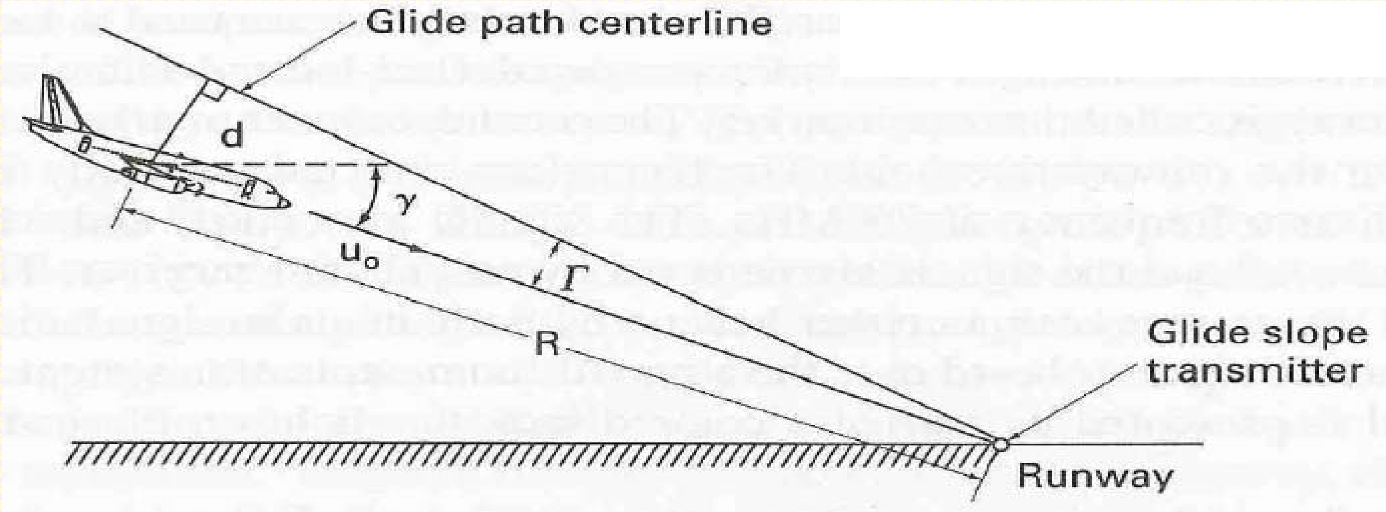
Alignment Control Through Electronic Aid and Pilot Input

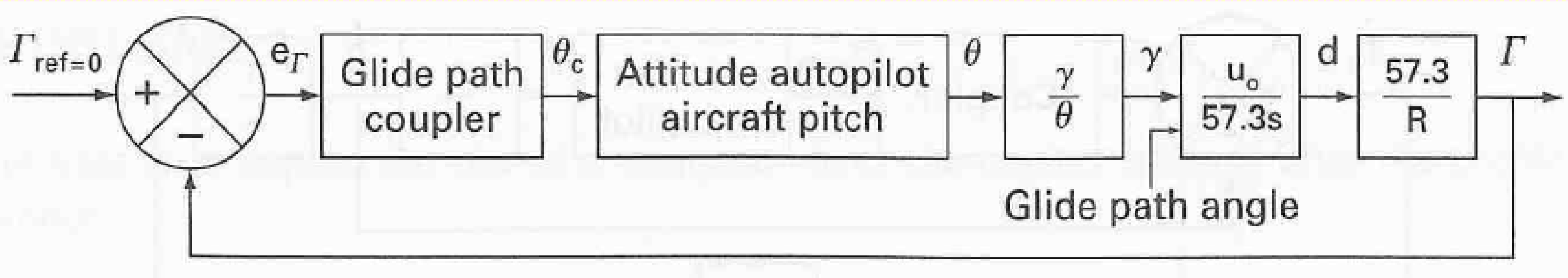
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.



Glide Slope: Pitch Control

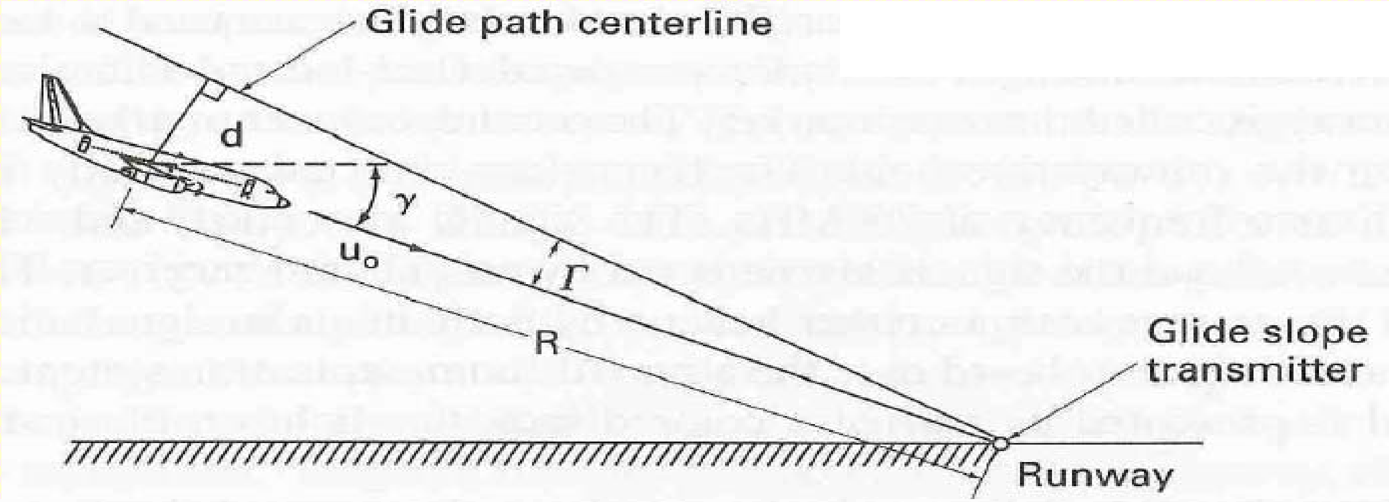
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

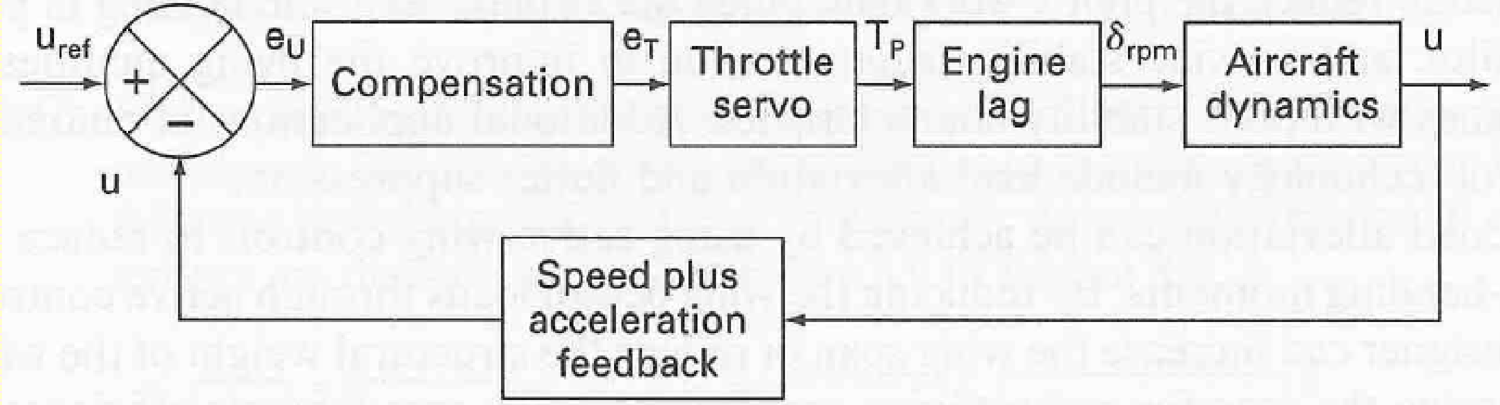




Glide Slope: Speed Control

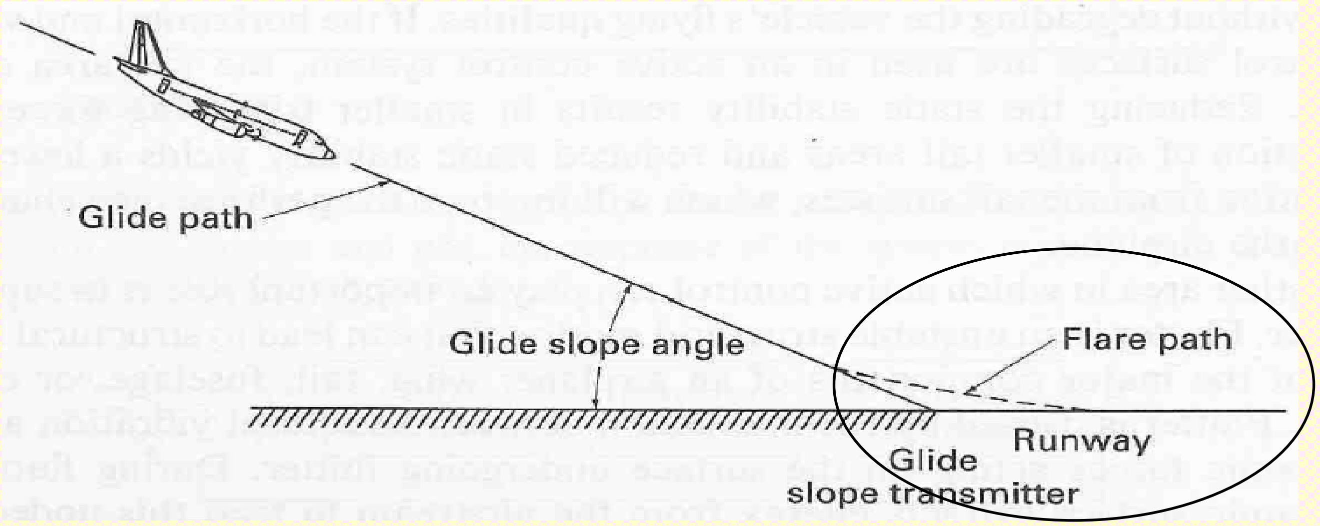
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

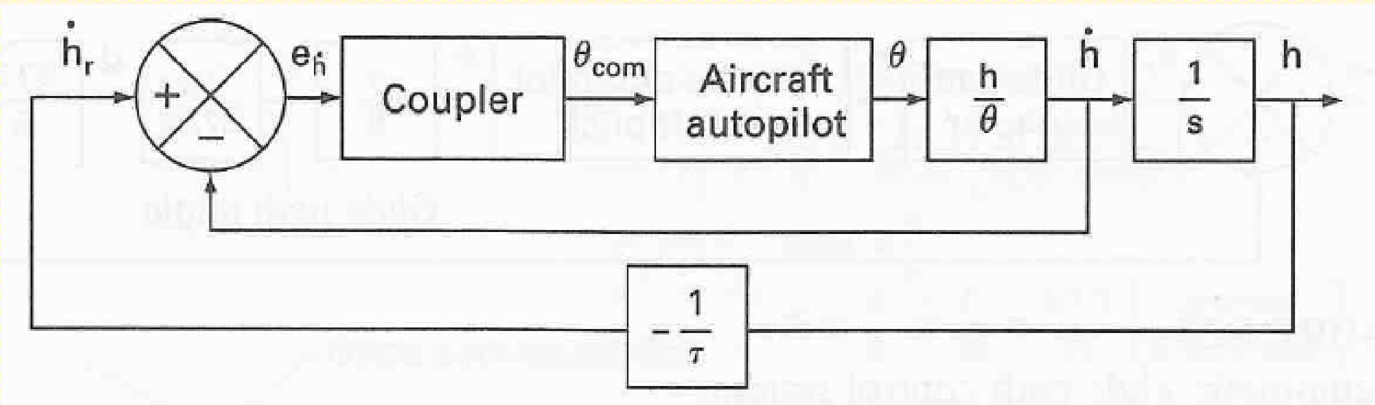




Flare: Sink Rate Control

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.





Thanks for the Attention...!