<u>Module I: Introduction and Motivation</u>

[Lectures: 1]

<u>Module II: Review of Classical Control</u>

[Lectures: 3, 4, 5, 6]

Assignment II.1. Find the forced response and natural response of a system given by the transfer function $G(s) = \frac{1}{s^2 + 2s}$ subjected to step input. What can you infer about the the stability of the system from the output response.

Assignment II.2. Solve the initial value problem represented by the differential equation $y'' - 4y' + 4y = t^2 e^t$; $y(0) = 0$; $y'(0) = 0$.

Assignment II.3. Find the transfer function of a RLC electric circuit (R-Resistance, L-Inductance, C-Capacitance) connected in series, subjected to an input of V_I Volts. The output is taken across the resistor, R as V_R. Take zero initial conditions.

Assignment II.4. Given the closed loop transfer function $T(s) = \frac{K}{s(s+5)(s-3)}$. Find the range of the gain, K for which the system becomes stable, unstable and marginally stable.

Assignment II.5. A system has infinite steady state error when subjected to a ramp input. Can you comment about the order of the system. Is it possible to bring the steady state error to a finite value? Suppose, our requirement is have a zero steady state error, how can it be done?

Assignment II.6. Draw the root locus plot for the open loop transfer function given as $G(s) = \frac{K(s+1)(s+2)}{(s-3)(s+4)(s+2)}$ and analyse the stability of the system as K varies. Find the values of K for which the system becomes unstable. What can you say about the damping ratio and the percentage overshoot for the stable system.

Module III: Flight Dynamics

[Lectures: 7, 11, 12]

Assignment III.1. Explain briefly about the skin friction drag and pressure drag. Also discuss the effect of different flows (laminar and turbulent flow) on the coefficient of drag.

Assignment III.2. Derive the force balance equation for a vehicle while pitching downward with an angle of attack of -20^0 and flight path angle of -5^0. The thrust is not acting along the body X-axis, but at an angle of 10^0 with respect to the velocity vector.

Assignment III.3. Derive the Six-DOF equations with respect wind axes frame.

Assignment III.4. Assignment II.4. Discuss about various ways of representing the attitudes of a rigid body relative to a reference frame.

Module IV: Representation of Linear Systems

[Lectures: 8, 9, 10]

Assignment IV.1. Find the state space form of the given differential equation $3\dddot{x}+4\ddot{x}+3\dot{x}+2x=5u$ and $y=\dot{x}+x+2u$. Discuss about the number of state variables required to completely describe the above system. Is the system stable?

Assignment IV.2. Verify the controllability and observability for the state model shown below,

$$\dot{X}=\begin{bmatrix}3 & 4\\ 2 & 0\end{bmatrix}X+\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}U$$

$$Y=\begin{bmatrix}-1 & 3\\ 1 & 4\end{bmatrix}X$$

Assignment IV.3. Consider the system shown below, compute its eigen values and eigen vectors

$$\dot{X}=\begin{bmatrix}3 & 4\\ 2 & 0\end{bmatrix}X$$

Module V: Review of the Matrix Theory

[Lectures: 13, 14, 15]

Assignment V.1. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find eigenvalues/eigenvectors of A. In addition, find the eigenvalues/eigenvectors of $A^{-1},\ A^m\ (m = 3, 4)\ and\ I + 2A + 4A^2$ without computing these matrices.

Assignment V.2. Prove $\left(AB\right)^{-1} = B^{-1}A^{-1}$, if the indicated matrix inverses exist.

Assignment V.3. An $n \times n$ Hadamard matrix A has all elements that are ± 1 and satisfies $A^T A = nI$ show that $\left|\det\left(A\right)\right| = n^{n/2}$

Assignment V.4. Show that the determinant of a negative definite $n \times n$ matrix is positive if n is even and negative if n is odd.

Assignment V.5. Let A and P be $n \times n$ matrices with P nonsingular.

Show that $Tr\left(A\right) = Tr\left(P^{-1}AP\right)$.

Assignment V.6. Following the standard definitions.

Show that for a fixed $X \in R^n, \left\|X\right\|_p \to \left\|X\right\|_\infty\ as\ \ p \to \infty$

Assignment V.7. Compute $\left\|A\right\|_1, \left\|A\right\|_2\ \left\|A\right\|_\infty\ and\ \ \rho\left(A\right)$ of the matrix in Assignment V.1.

Assignment V.8. Find the singular value decomposition of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Give all the steps. Verify the result using MATLAB.

Assignment V.9. Show that $\frac{\partial}{\partial X}\left(\frac{1}{2}X^T AX\right) = AX$.

Assignment V.10. Show that if

$$F\left(f\left(X\right)\right) \in R^n,\ X \in R^n$$

$$then\ \left[\frac{\partial F}{\partial X}\right] = \left[\frac{\partial F}{\partial f}\right]^T \left[\frac{\partial F}{\partial X}\right]_{n\times n}$$

Assignment V.11. Show that if a real matrix has one eigenvalue as complex then its conjugate is also an eigen value.

Module VI: Review of Numerical Methods

[Lectures: 16]

Assignment VI.1. Find the roots of the equation

$$f(x) = x - \cos(x)$$

$x(0) = 0.5$ and accuracy = 10^{-5}

by Newton-Raphson method.

Assignment VI.2. Solve the below given differential equation by RK-4 method from t = 0 to t = 400 min with given initial conditions. Use $\Delta t = 1\text{min}$.

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{Bmatrix} -p_1 x_1 - (x_1 + G_b)\, x_2 + x_4 \\ -p_2 x_2 + p_3 x_3 \\ -n x_3 \\ -B x_4 \end{Bmatrix}$$

$$\begin{aligned} x_1(0) &= 100 \\ x_2(0) &= 0 \\ x_3(0) &= 10 \\ x_4(0) &= 10 \end{aligned}$$

$$\begin{aligned} p_1 &= 0 \\ p_2 &= 0.0142 \\ p_3 &= 1.54\times10^{-5} \\ B &= 0.05 \\ n &= 0.2814 \\ G_b &= 70 \\ I_b &= 7 \end{aligned}$$

<u>Module VII: Linearization of Nonlinear Systems</u>

[Lectures: 17]

Assignment VII.1. Linearize the system $\dot{x} = x \; -$ about its equilibrium point/s.

Module VIII: Time Response, Stability, Controllability and Observability of Linear Systems

[Lectures: 2, 18, 19, 20]

Assignment VIII.1. If second order system has two poles at origin then it is unstable, prove.

Assignment VIII.2. Compute state transition matrix and transfer function for the state model shown below:

$$\dot{X} = \begin{bmatrix} -3 & 4 \\ 4 & 10 \end{bmatrix} X + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} X$$

Module IX: Pole Placement, Controller and Observer Design of Linear Systems

[Lectures: 21, 22]

Assignment IX.1. Consider the linear system

$$\dot{X} = AX + Bu$$
$$y = CX$$

where, $A = \begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 \end{bmatrix}$.

Assume that the output y can be accurately measured. Design reduced order observer assuming that the desired eigen value is $\mu = -4$.

Module X: Static Optimization

[Lecture: 23]

Module XI: Optimal Control Design

[Lectures: 24, 25, 26, 27, 28]

Assignment XI.1. Solve the following problem using Kuhn-Tucker conditions. Also check for sufficiency condition.

Minimize: $J(X) = \left(x_1^2 + x_2^2\right)$

Subject to: $x_1 + x_2 \leq 9$

$x_1 + x_2 \geq 0$

Assignment XI.2. Show that shortest distance between two points in a plane is straight line using calculus of variation.

Assignment XI.3. Let us consider the system:

$$\dot{x}(t) = -2x(t) + 2u(t) \qquad \text{with } x(0) = 1$$

and the cost function as $J = \frac{1}{2}\int_0^\infty e^{2\alpha t}\left[x^2(t) + 4u^2(t)\right]$

Find the optimal control law. Also show that the closed-loop optimal system has a degree of stability of at least α.

Module XII: Linear Control Applications in Flight Control Design

[Lectures: 29, 30, 38]

Assignment XII.1. Design a stability augmentation system having a poor short-period flying qualities in a particular flight regime. Determine the feedback gains so that the airplane short-period characteristics are $\lambda_{sp} = -2.1 \pm 2.14j$. Assume that the original short period dynamics are given by:

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} -0.334 & 1.0 \\ -2.52 & -0.387 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} -0.0.27 \\ -2.6 \end{bmatrix} \Delta\delta_e$$

Module XIII: Nonlinear System Analysis Using Lyapunov Theory

[Lectures: 33, 34, 35]

Assignment XIII.1. If a constant symmetric matrix P satisfies linear matrix (Lyapunov equation)

$$AP + PA^T + Q = 0$$

Show that $P = \int_0^\infty e^{A^T t} Q e^{At} dt$.

Assignment XIII.2. Consider the system defined by

$$\dot{x}_1 = \frac{2}{3}x_2 + x_1\left(\alpha^2 - 3x_1^2 - 2x_2^2\right)$$
$$\dot{x}_1 = -x_1 + x_2\left(\alpha^2 - 3x_1^2 - 2x_2^2\right)$$

Show that the origin and limit cycle $3x_1^2 + 2x_2^2 = \alpha^2$ are invariant sets. Study the stability of the origin and the limit cycle using Lyapunov theory and LaSalle's theorem.

Assignment XIII.3. Study the stability of the equilibrium point at origin of the following system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\beta x_1 + x_1^2 x_2$$

Module XIV: Nonlinear Control Synthesis

[Lectures: 31, 32, 36, 37]

Module XV: Nonlinear Observer and Kalman Filter Design

[Lectures: 39, 40]

Assignment XV.1. Consider the following system

$$\dot{x}_1 = x_1 + \sin x_1 + x_2$$
$$\dot{x}_2 = u$$

Using backstepping approach, design a state feedback law to stabilize the equilibrium point at the origin.

Assignment XV.2. Consider the following system:

$$\dot{x}(t) = ax(t) + gw(t)$$
$$y(t) = cx(t) + v(t)$$

Where, a, c and g are scalar constant, the standard deviations of the zero-mean white Gaussian noises $w(t)$ and $v(t)$ are given by q and r respectively. Design the Kalman filter.