TURBOMACHINERY AERODYNAMICS

Lect- 33

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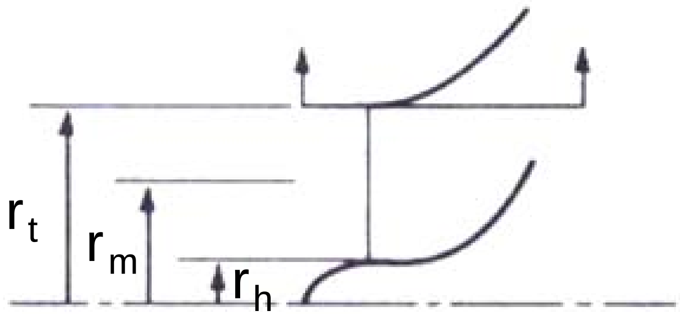
In this lecture...

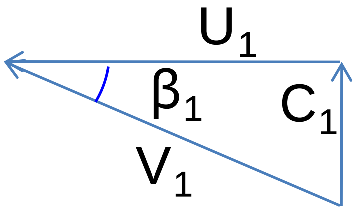
- Tutorial on centrifugal compressors

Problem # 1

- At the inlet of a centrifugal compressor eye, the relative Mach number is to be limited to 0.97. The hub-tip radius ratio of the inducer is 0.4. The eye tip diameter is 20 cm. If the inlet velocity is axial, determine, (a) the maximum mass flow rate for a rotational speed of 29160 rpm, (b) the blade angle at the inducer tip for this mass flow. The inlet conditions can be taken as 101.3 kPa and 288 K.

Problem # 1





Solution: Problem # 1

The rotational speed at the inducer tip is

$$U_1 = \pi dN / 60 = \pi \times 0.2 \times 29160 / 60 = 305.36 m/s$$

From the velocity traingle, we can see that

$$M_{1rel} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R T_1}}$$

$$T_1 = T_{01} - C_1^2 / 2c_P = 288 - C_1^2 / 2010$$

$$M_{1rel} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R (288 - C_1^2 / 2010)}}$$

$$0.97^2 = \frac{C_1^2 + 305.63^2}{115718.4 - 0.2 C_1^2}$$

Simplifying, $C_1 = 114.62 m/s$

Solution: Problem # 1

$$T_1 = T_{01} - C_1^2 / 2c_P = 288 - C_1^2 / 2010 = 281.464K$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)}$$

Substituting, $P_1 = 93.48kPa$

$\therefore \rho_1 = P_1 / RT_1 = 1.157kg / m^3$

Annulus area at the inlet, $A_1 = \frac{\pi}{4} d^2 (1 - r_h / r_t)$

$A_1 = 0.0264m^2$

Solution: Problem # 1

Since the flow is axial,

$$C_{a1} = C_1$$

$$\therefore \dot{m} = \rho_1 A_1 C_1 = 1.157 \times 0.0264 \times 114.62 = 3.5 \text{kg/s}$$

The blade inlet angle at the tip is

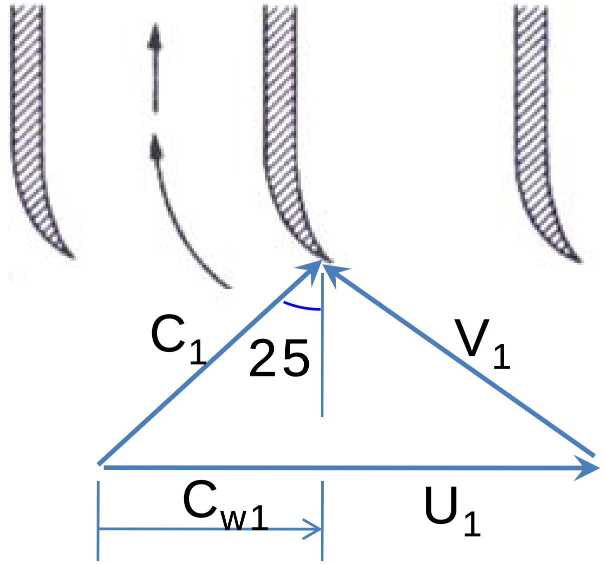
$$\tan\beta_1 = C_1 / U_1$$

$$\therefore \beta_1 = 20.57^o$$

Problem # 2

- A centrifugal compressor has a pressure ratio of 4:1 with an isentropic efficiency of 80% when running at 15000 rpm and inducing air at 293 K. Curved vanes at the inlet give the air a pre-whirl of 25^o to the axial direction at all radii. The tip diameter of the eye of the impeller is 250 mm. The absolute velocity at inlet is 150 m/s and the impeller diameter is 600 mm. Calculate the slip factor.

Problem # 2



Solution: Problem # 2

Exit stagnation temperature is

$$T_{02} = T_{01}(\pi_c)^{(\gamma-1)/\gamma} = 293(4)^{(1.4-1)/1.4} = 435.56K$$

Therefore the isentropic temperature rise,

$$\Delta T_{0s} = 435.56 - 293 = 142.56K$$

The actual temperature rise, $\Delta T_0 = \Delta T_{0s} / \eta_c$

$$\Delta T_0 = 178.2K$$

Work done per unit mass is, $w = c_P \Delta T_0$

$$w = 1.005 \times 178.2 = 179 \text{ kJ/kg}$$

Solution: Problem # 2

Peripheral velocity at the tip of the eye,

$$U_1 = \pi dN / 60 = \pi \times 0.25 \times 15000 / 60 = 196.25 m/s$$

$$C_{w1} = C_1 \sin 25 = 63.4 m/s$$

Peripheral velocity at the tip of the impeller,

$$U_2 = \pi DN / 60 = \pi \times 0.60 \times 15000 / 60 = 471.2 m/s$$

Solution: Problem # 2

We know that power input is, $w = U_2 C_{w2} - U_1 C_{w1}$

$$179 \times 10^3 = 471.24 \times C_{w2} - 196.35 \times 63.4$$

or, $C_{w2} = 406.27 m/s$

Therefore, the slip factor is,

$$\sigma_s = C_{w2} / U_2 = 0.862$$

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Problem # 3

Air at a stagnation temperature of 22°C enters the impeller of a centrifugal compressor in the axial direction. The rotor, which has 17 radial vanes, rotates at 15,000 rpm. The stagnation pressure ratio between diffuser outlet and impeller inlet is 4.2 and total-to-total efficiency is 83%. Determine the impeller tip radius. Assume the air density at impeller outlet is $2 kg/m^3$ and the axial width at entrance to the diffuser is 11mm, determine the absolute Mach number at that point. Assume that the slip factor $\sigma = 1 - 2/N$, where N is the number of vanes.

Problem # 3

The specific work required is

$$w_c = U_2 C_{w2} - U_1 C_{w1}$$

Since $C_{w1} = 0$, $w = U_2 C_{w2} = \sigma U_2^2$

Expressing U_2 in terms of efficiency and pressure ratio,

$$U_2^2 = \frac{c_p T_{01}(\pi_c^{(\gamma-1)/\gamma} - 1)}{\sigma \eta_{c-tt}}$$

$$\sigma = 1 - 2/N = 1 - 2/17 = 0.8824$$

Substituting all other values, $U_2 = 452 \text{ m/s}$

Since, $\Omega = 15000 \times 2\pi / 60 = 1570 \text{ rad/s}$

Therefore, the impeller radius is

$$r_t = U_2 / \Omega = 0.288 \text{ m}$$

Problem # 3

Mach number, $M_2 = C_2 / a_2 = C_2 / \sqrt{\gamma R T_2}$

where, $C_2 = \sqrt{C_{w2}^2 + C_{r2}^2}$

$C_{r2} = \dot{m} / (\rho_2 2\pi r_t b_2) = 2 / (2 \times 2\pi \times 0.288 \times 0.011) = 50.3\,m/s$

$C_{w2} = \sigma U_2 = 400\ m/s$

$\therefore C_2 = \sqrt{50.3^2 + 400^2} = 402.5\ m/s$

We know that $h_{02} = h_{01} + w_c = h_{01} + \sigma U_2^2$

$$\text{or,}\quad h_2 = h_{01} + \sigma U_2^2 - \tfrac{1}{2} C_2^2$$

$$\text{or,}\quad T_2 = T_{01} + (\sigma U_2^2 - \tfrac{1}{2} C_2^2) / c_p$$

$$= 394.5\ K$$

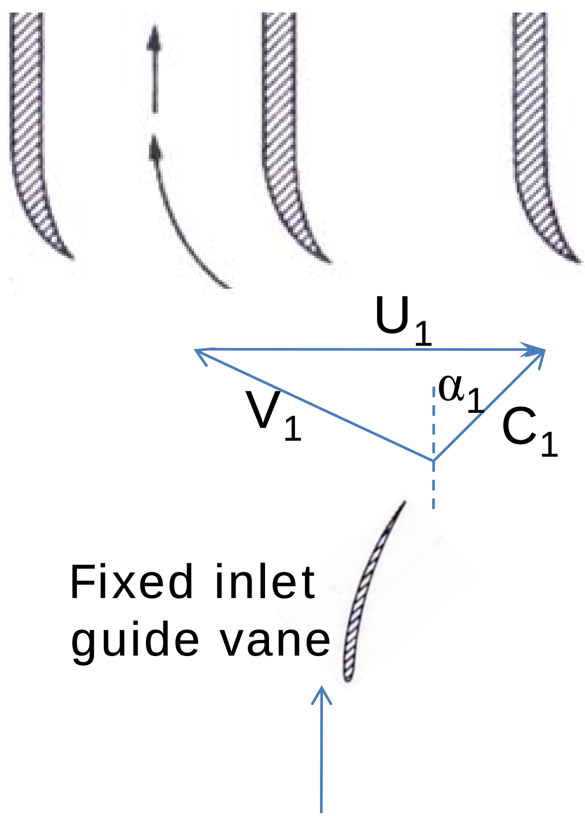
Therefore, $M_2 = 402.5 / \sqrt{1.4 \times 287 \times 394.5} = 1.01$

Problem # 4

A centrifugal compressor with backward leaning blades develops a pressure ratio of 5:1 with an isentropic efficiency of 83 percent. The compressor runs at 15000 rpm. Inducers are provided at the inlet of the compressor so that air enters at an absolute velocity of 120 m/s. The inlet stagnation temperature is 250 K and the inlet air is given a pre-whirl 22^o to the axial direction at all radii. The mean diameter of the eye of the impeller is 250 mm and the impeller tip diameter is 600 mm. Determine the slip factor and the relative Mach number at the impeller tip.

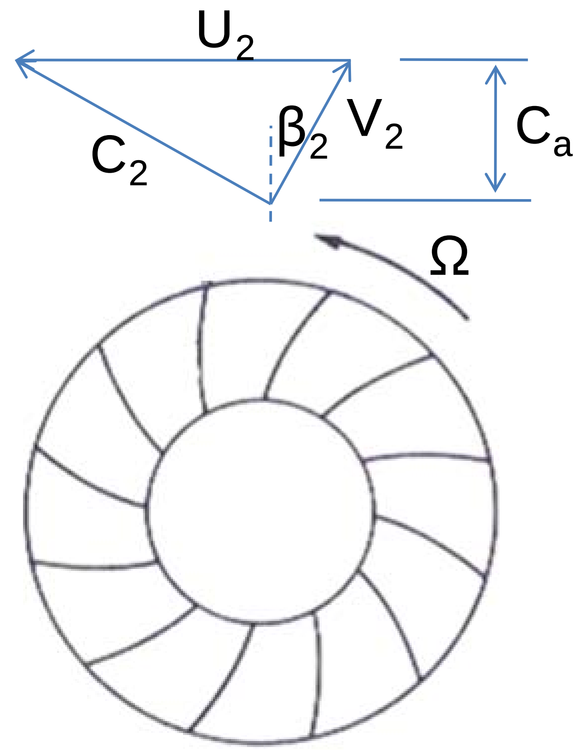
Problem # 4

Inducer



Fixed inlet guide vane

Inlet velocity triangle



Exit velocity triangle

Solution: Problem # 4

$T_{01} = 300\ K$

$T_{02s} = T_{01}(\pi_c)^{(\gamma-1)/\gamma} = 250\,(5)^{0.4/1.4} = 395.95\ K$

$\Delta T_{0s} = 395.95 - 300 = 95.95\ K$

Actual temperature rise, $\Delta T_{0actual} = \Delta T_{0s} / \eta_c = 95.95 / 0.83$

$= 115.6\,K$

The specific work required, $w_c = c_p \Delta T_{0actual} = 1005 \times 115.6$

$= 116.186\ kJ/kg$

Given that $C_1 = 150\,m/s$, $\therefore C_{w1} = C_1 \sin\alpha_1 = 150 \sin 22$

$= 56.2\,m/s$

Solution: Problem # 4

$$U_1 = \pi d_m N / 60 = \pi \times 0.25 \times 15000 / 60 = 196.3 \text{ m/s}$$

$$\text{and } U_2 = \pi d_t N / 60 = \pi \times 0.6 \times 15000 / 60 = 471.24 \text{ m/s}$$

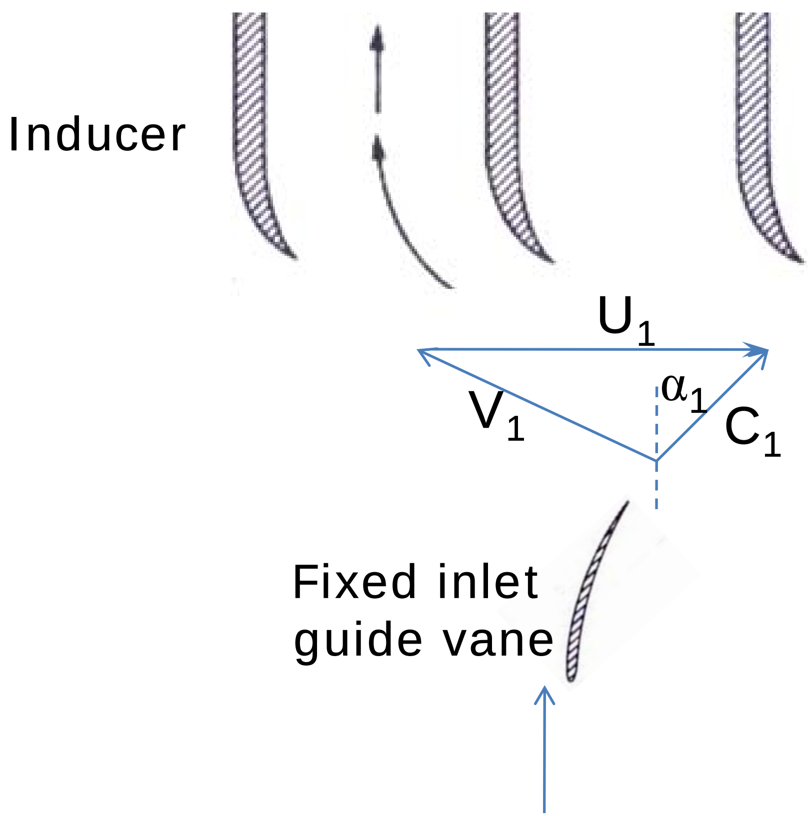
$$\text{Since, } w_c = U_2 C_{w2} - U_1 C_{w1}$$

$$116.186 \times 10^3 = 471.24 \times C_{w2} - 196.3 \times 56.2$$

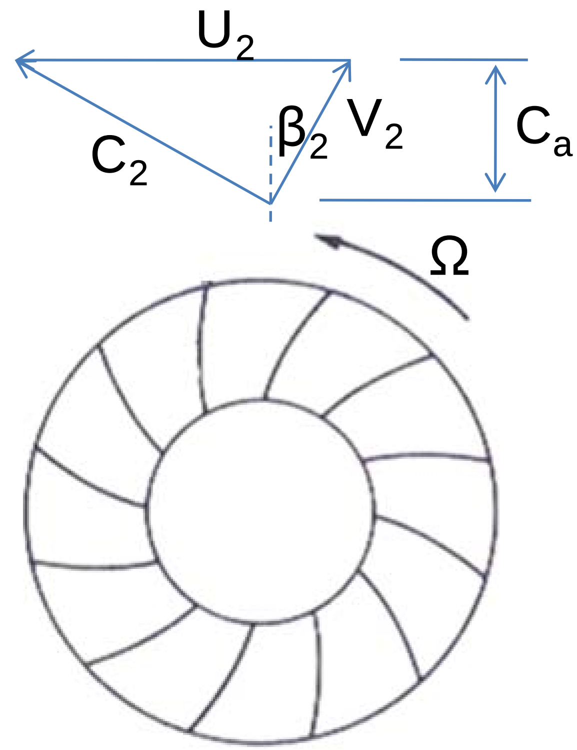
$$\therefore C_{w2} = 269.96 \text{ m/s}$$

$$\text{The slip factor, } \sigma = C_{w2} / U_2 = 269.96 / 471.24 = 0.573$$

Problem # 4



Inlet velocity triangle



Exit velocity triangle

Solution: Problem # 4

From the impeller exit velocity triangle,

$$V_2 = \sqrt{C_a^2 + (U_2 - C_{w2})^2} = \sqrt{(C_1 \cos\alpha_1)^2 + (U_2 - C_{w2})^2}$$

$$= 222.9 \text{ m/s}$$

$$M_{rel} = V_2 / \sqrt{\gamma R T_2}$$

$$T_2 = T_{02} - C_2^2 / 2c_p$$

$$T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_c} = 365.61\text{K}$$

and $C_2 = \sqrt{C_{w2}^2 + C_a^2} = \sqrt{269.9^2 + 139.08^2} = 303.68 \text{ m/s}$

$$\therefore T_2 = 365.61 - 303.68^2 / 2 \times 1005 = 319.73\text{K}$$

The relative Mach number at the impeller tip is

$$M_{rel} = 222.9 / \sqrt{(1.4 \times 287 \times 319.73)} = 0.62$$