TURBOMACHINERY AERODYNAMICS

Lect- 23

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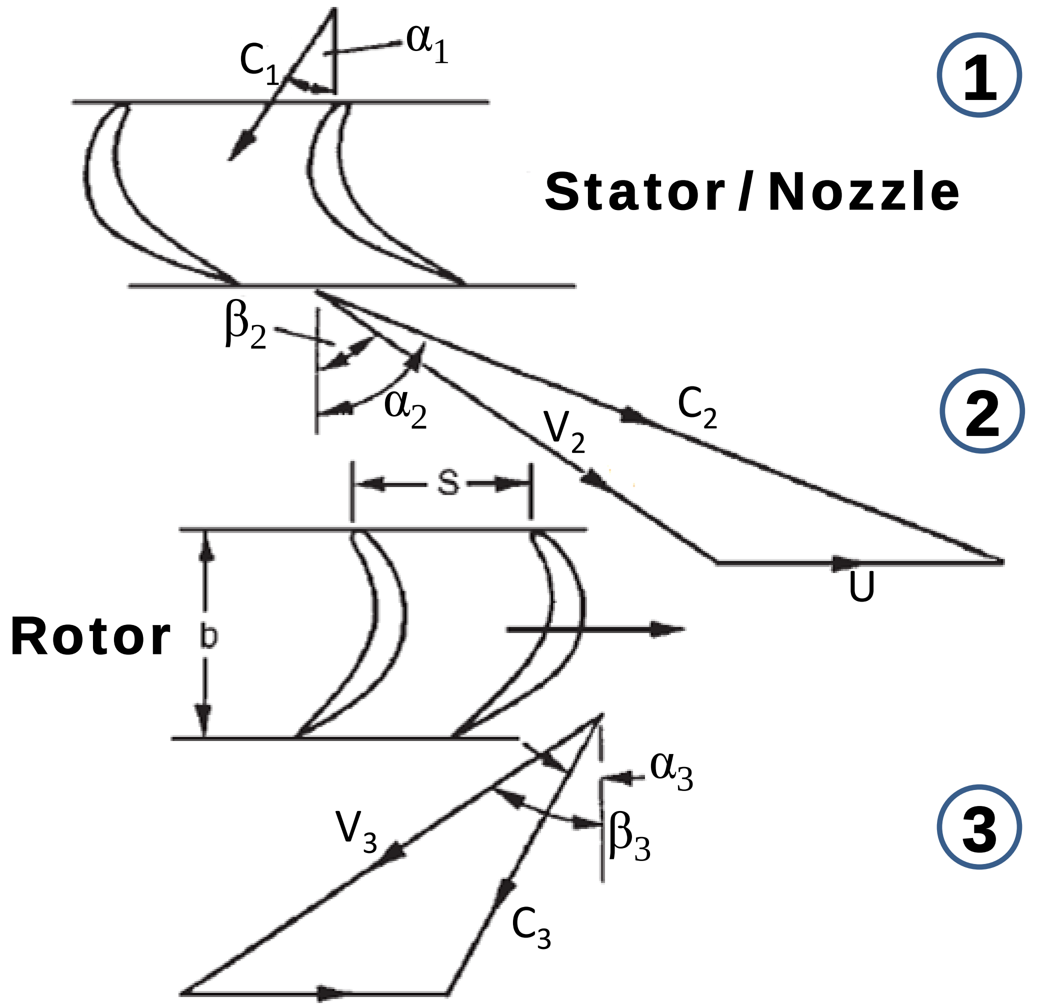
In this lecture...

- Tutorial on axial flow turbines

Problem # 1

- A single stage gas turbine operates at its design condition with an axial absolute flow at entry and exit from the stage. The absolute flow angle at the nozzle exit is 70 deg. At stage entry, the total pressure and temperature are 311 kPa and 850^{o}C respectively. The exhaust static pressure is 100 kPa, the total to static efficiency is 0.87 and mean blade speed is 500 m/s. Assuming constant axial velocity through the stage, determine (a) the specific work done (b) the Mach number leaving the nozzle (c) the axial velocity (d) total to total efficiency (e) stage reaction.

Problem # 1



Solution: Problem # 1

We know that total – to – static efficiency,

$$\eta_{ts} = \frac{w_t}{c_p T_{01}\left[1-(P_3/P_{01})^{(\gamma-1)/\gamma}\right]}$$

$$\therefore \text{ Specific work is, } w_t = \eta_{ts} c_p T_{01}\left[1-(P_3/P_{01})^{(\gamma-1)/\gamma}\right]$$

$$= 0.87 \times 1148 \times 1123 \times \left[1-(1/3.11)^{0.248}\right]$$

$$= 276\, \text{kJ/kg}$$

(b) At the nozzle exit, the Mach number is

$$M_2 = C_2 / \sqrt{\gamma R T_2}$$

From the velocity triangle, $C_{w3} = 0$, $w_t = UC_{w2}$

$$\therefore\ C_{w2} = w_t / U = 276 \times 10^3 / 500 = 552\, \text{m/s}$$

Solution: Problem # 1

$C_2 = C_{w2} / \sin\alpha_2 = 588 \text{ m/s}$

We know that $T_2 = T_{01} - \frac{1}{2}C_2^2 / c_p = 973\text{K}$

Hence, $M_2 = 588 / \sqrt{1.33 \times 287 \times 973} = 0.97$

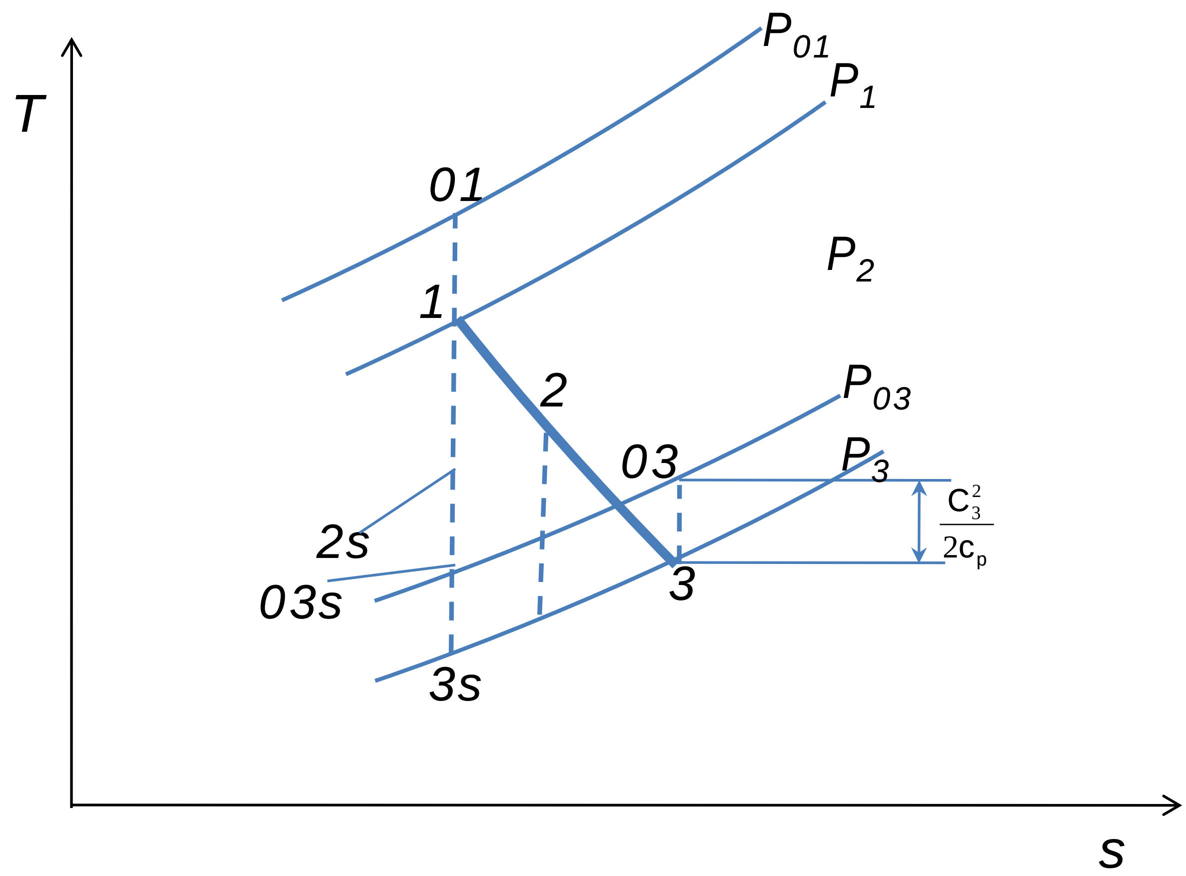
(c) The axial velocity, $C_a = C_2 \cos\alpha_2 = 200 \text{ m/s}$

(d) The total - to - total efficiency is related to the total - to - static efficiency as :

$$\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2w_t} = \frac{1}{0.87} - \frac{200^2}{2 \times 276 \times 10^3} = 1.0775$$

$$\therefore \eta_{tt} = 0.93$$

Solution: Problem # 1



Expansion process in a turbine stage

Solution: Problem # 1

(e) Degree of reaction, $R_x = 1 - \frac{1}{2}(C_a / U)(\tan\beta_3 - \tan\beta_2)$

From the velocity triangle,

$\tan\beta_3 = U / C_a$ and $\tan\beta_2 = \tan\alpha_2 - U / C_a$

$\therefore R_x = 1 - \frac{1}{2}(C_a / U)\tan\alpha_2$

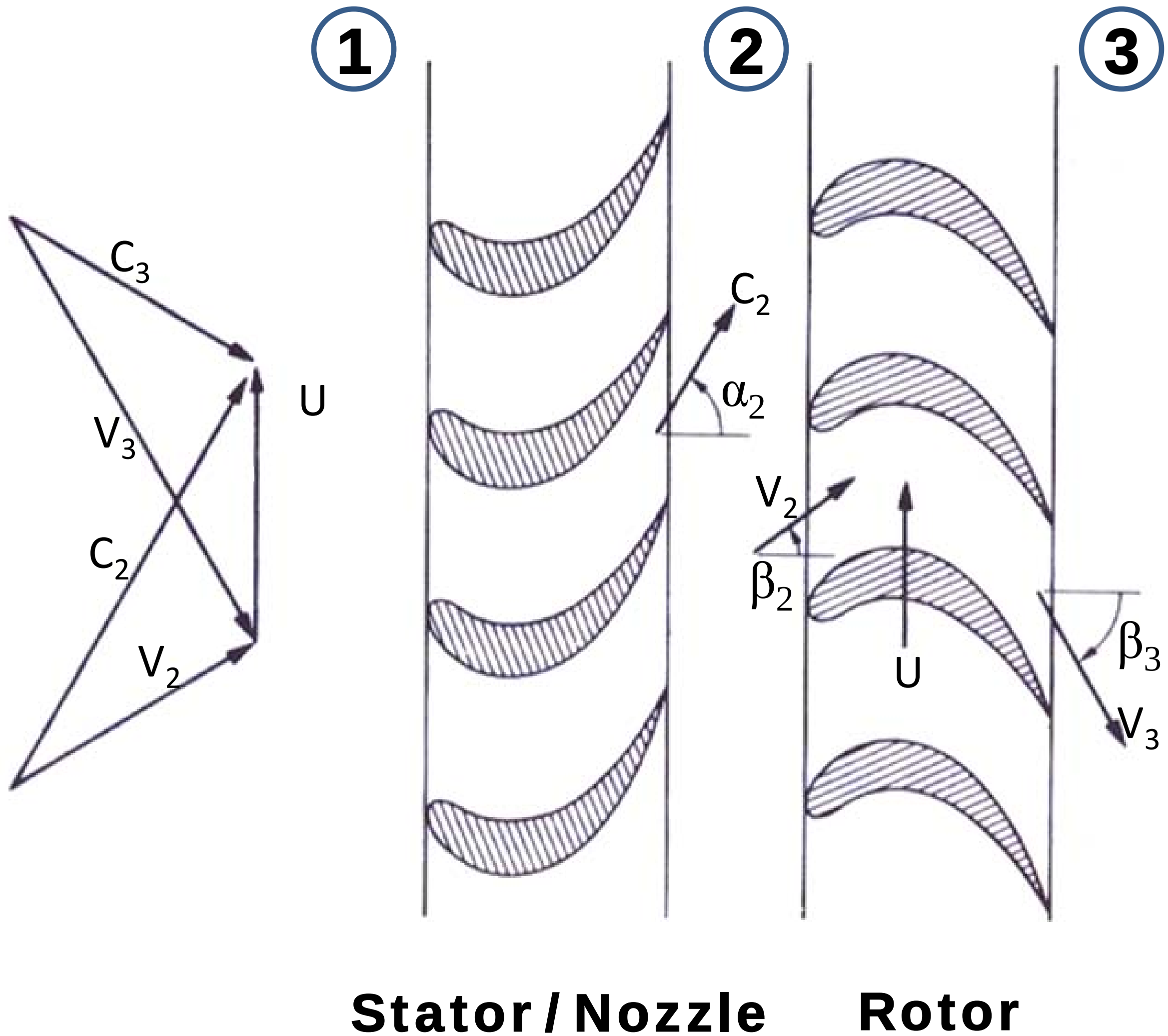
$= 1 - 200 \times 0.2745 / 1000$

$= 0.451$

Problem # 2

Combustion gases enter the first stage of a gas turbine at a stagnation temperature and pressure of 1200 K and 4.0 bar. The rotor blade tip diameter is 0.75m, the blade height is 0.12 m and the shaft speed is 10,500 rpm. At the mean radius the stage operates with a reaction of 50%, a flow coefficient of 0.7 and a stage loading coefficient of 2.5. Determine (a) the relative and absolute flow angles for the stage; (b) the velocity at nozzle exit; (c) the static temperature and pressure at nozzle exit assuming a nozzle efficiency of 0.96 and the mass flow.

Solution: Problem # 2



Solution: Problem # 2

(a) The stage loading is given by

$\psi = \Delta h_0 / U^2 = (V_{w3} + V_{w2}) / U = (C_a / U)(\tan\beta_3 + \tan\beta_2)$

Also, $R_x = (C_a / U)(\tan\beta_3 - \tan\beta_2) / 2$

Simplifying the above equations,

$\tan\beta_3 = (\psi / 2 + R) / (C_a / U)$ and $\tan\beta_2 = (\psi / 2 - R) / (C_a / U)$

Substituting values of ψ, $(C_a / U) = \varphi$ and R_x

$\beta_3 = 68.2^o$ and $\beta_2 = 46.98^o$

For a 50% reaction stage, $\alpha_2 = \beta_3 = 68.2^o$ and $\alpha_3 = \beta_2 = 46.98^o$

Solution: Problem # 2

(b) At the mean radius, $r_m = (0.75 - 0.12) / 2 = 0.315\text{m}$

the blade speed, $U_m = (10500 / 30) \times \pi \times 0.315 = 346.36 \text{ m/s}$

The axial velocity, $C_a = \varphi U_m = 242.45 \text{ m/s}$ and

Therefore, velocity at the nozzle exit,

$C_2 = C_a / \cos\alpha_2 = 242.45 / \cos 68.2 = 652.86 \text{ m/s}$

(c) The static temperature at the nozzle exit,

$T_2 = T_{02} - C_2^2 / 2c_p = 1200 - 652.86^2 / (2 \times 1160) = 1016.3\text{K}$

Solution: Problem # 2

The nozzle efficiency, $\eta_n = \dfrac{h_{01} - h_2}{h_{01} - h_{2s}} = \dfrac{1 - T_2 / T_{01}}{1 - (P_2 / P_{01})^{(\gamma-1)/\gamma}}$

$$\therefore (P_2 / P_{01})^{(\gamma-1)/\gamma} = 1 - \frac{1 - T_2 / T_{01}}{\eta_n} = 0.84052$$

$$\therefore P_2 = 4 \times 0.84052^{4.0303} = 1.986 \text{ bar}$$

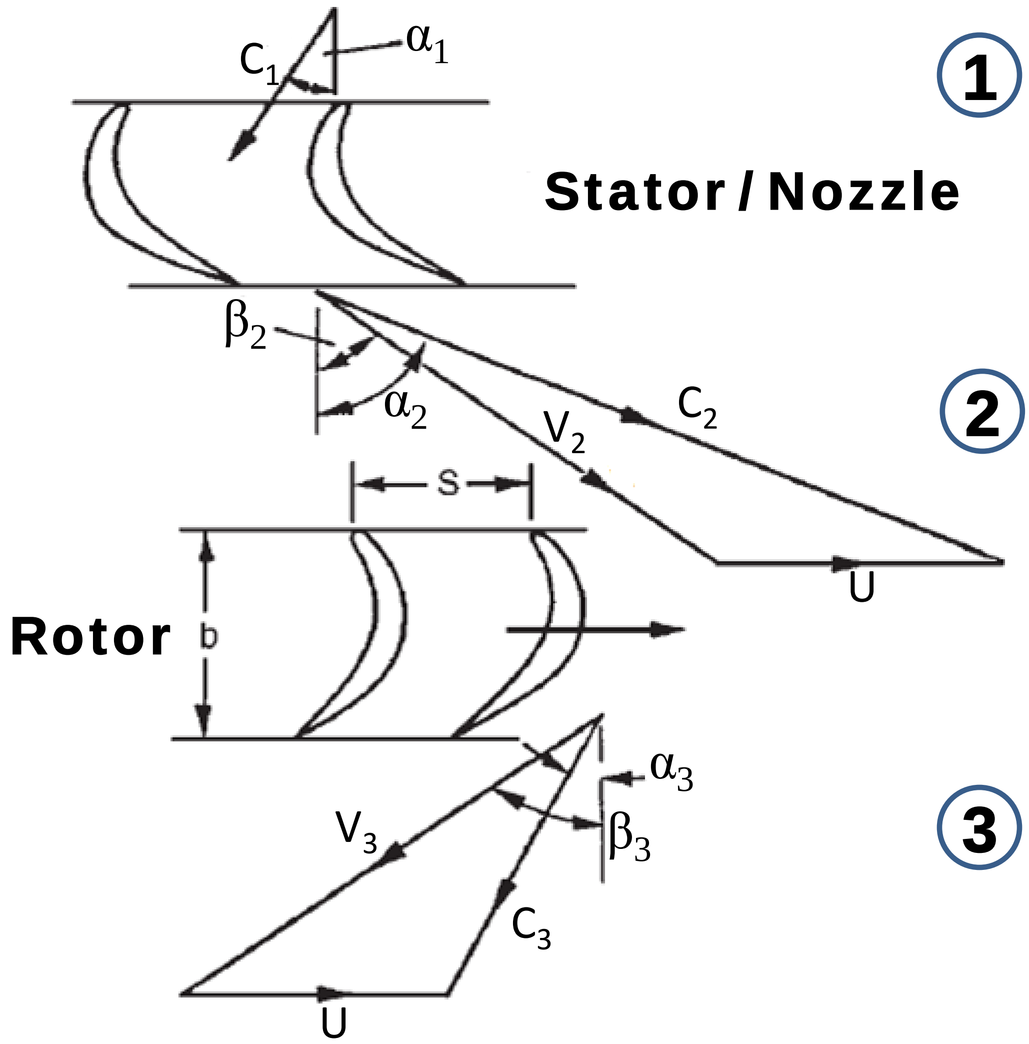
The mass flow rate is $\dot{m} = \rho_2 A_2 C_a = (P_2 / RT_2) A_2 C_a$

$$\therefore \dot{m} = (1.986 \times 10^5 / 287 \times 1016.3) \times 0.2375 \times 242.45 = 39.1 \text{ kg/s}$$

Problem # 3

A single stage axial flow turbine operates with an inlet temperature of 1100 K and total pressure of 3.4 bar. The total temperature drop across the stage is 144 K and the isentropic efficiency of the turbine is 0.9. The mean blade speed is 298 m/s and the mass flow rate is 18.75 kg/s. The turbine operates with a rotational speed of 12000 rpm. If the convergent nozzle is operating under choked condition determine (a) blade-loading coefficient (b) pressure ratio of the stage and (c) flow angles.

Problem # 3



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Problem # 3

(a) The blade loading is defined as

$$\psi = \frac{c_p \Delta T_0}{U^2} = \frac{1148 \times 144}{298^2} = 1.8615$$

(b) $T_{02} = T_{01} = 1100\,K$

$$T_{03} = T_{01} - \Delta T_0 = 1100 - 144 = 956\,K$$

The isentropic efficiency of a turbine, $\eta_t = \dfrac{T_{01} - T_{03}}{T_{01} - T_{03s}}$

$$= \frac{\Delta T_0}{T_{01}\left[1 - (P_{03} / P_{01})^{(\gamma-1)/\gamma}\right]}$$

or $\dfrac{P_{03}}{P_{01}} = \left[1 - \dfrac{\Delta T_0}{\eta_t T_{01}}\right]^{\gamma/(\gamma-1)} = 0.533$

The pressure ratio of the turbine is

$\dfrac{P_{01}}{P_{03}} = 1.875$ and $P_{03} = 1.813$ bar

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Problem # 3

(c) Since the nozzle is choked, the exit Mach number is unity.

Therefore, $C_2 = \sqrt{\gamma R T_2}$

and $\frac{T_{02}}{T_2} = \frac{\gamma + 1}{2} = 1.165$

The static temperature at the nozzle exit is $T_2 = 944.2\,K$.

The absolute velocity of the gases leaving the choked nozzle is therefore, $C_2 = 600.3\,m/s$.

The axial velocity $C_a = U\varphi = 298 \times 0.95 = 283\,m/s$.

From the velocity triangles,

$\cos\alpha_2 = C_a / C_2 = 283/600 = 0.4716$ and $\alpha_2 = 62^o$

Problem # 3

$$\frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 = \frac{1}{\varphi}$$

$$\tan\beta_2 = \tan\alpha_2 - \frac{1}{\varphi} = 0.828 \quad \text{or} \quad \beta_2 = 39.6^o$$

The turbine specific work, $w_t = c_p\Delta T_o = UC_a(\tan\alpha_2 + \tan\alpha_3)$

$$\text{or} \quad \tan\alpha_3 = \frac{c_p\Delta T_o}{UC_a} - \tan\alpha_2 = \frac{1148 \times 144}{298 \times 283} - 1.8807 = 0.0793$$

$$\text{or} \quad \alpha_3 = 4.54^o$$

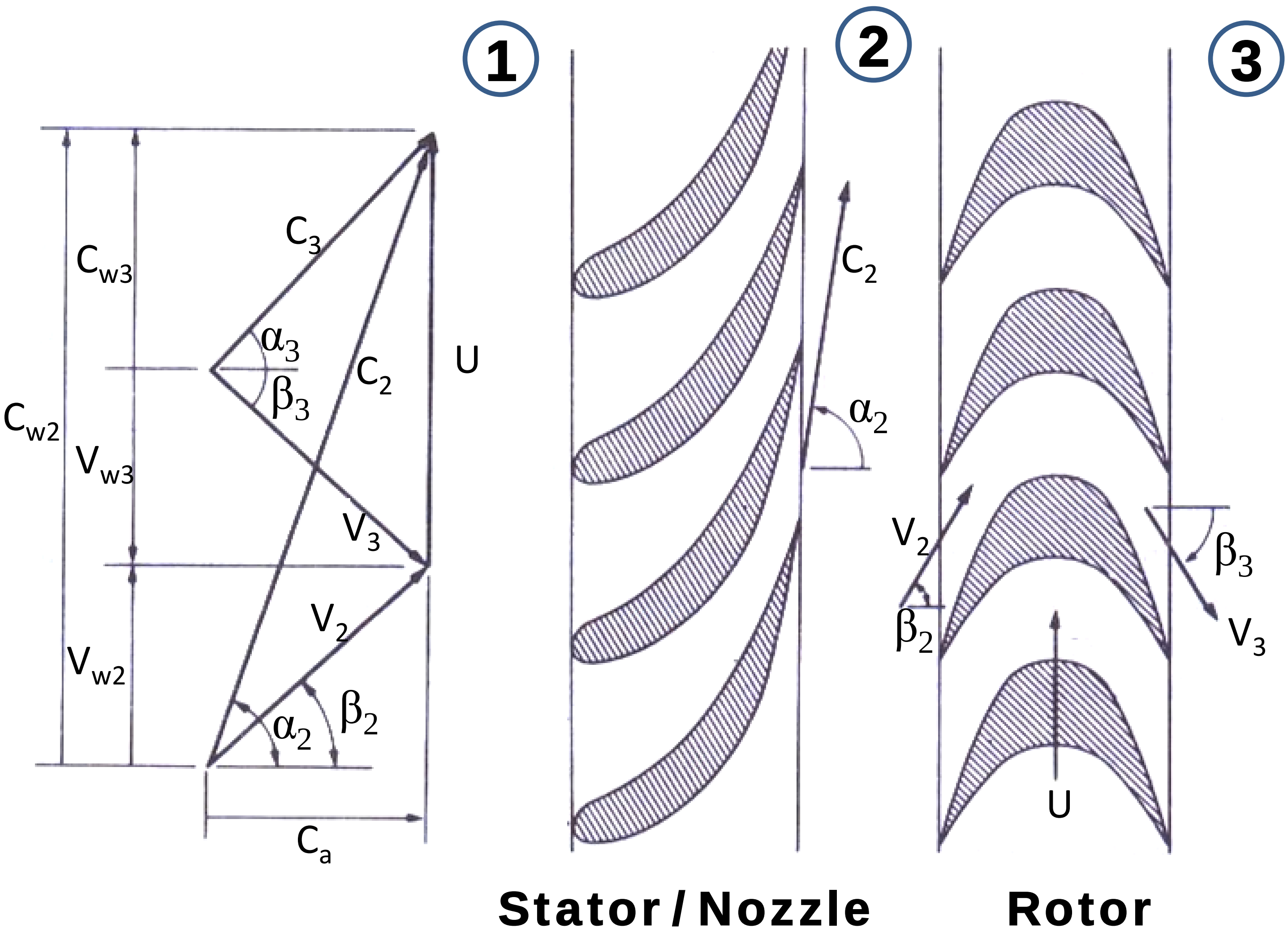
$$\frac{U}{C_a} = \tan\beta_3 - \tan\alpha_3 \quad \text{or} \quad \tan\beta_3 = \frac{1}{\varphi} + \tan\alpha_3 = 1.132$$

and therefore, $\beta_3 = 48.54^o$

Problem # 4

A multi-stage axial turbine is to be designed with impulse stages and is to operate with an inlet pressure and temperature of 6 bar and 900 K and outlet pressure of 1 bar. The isentropic efficiency of the turbine is 85 %. All the stages are to have a nozzle outlet angle of 75^o and equal inlet and outlet rotor blade angles. Mean blade speed is 250 m/s and the axial velocity is 150 m/s and is a constant across the turbine. Estimate the number for stages required for this turbine.

Problem # 4


1
2
3
C_{w3}
C_3
α_3
β_3
C_2
U
C_{w2}
V_{w3}
V_3
V_2
V_{w2}
α_2
β_2
C_a
C_2
α_2
V_2
β_2
β_3
V_3
U
Stator / Nozzle
Rotor

Solution: Problem # 4

Since the overall pressure ratio is known,

$$\frac{T_{01}}{T_{0es}} = \left(\frac{P_{01}}{P_{0e}}\right)^{(\gamma-1)/\gamma} = 6^{0.33/1.33}$$

$$\therefore\ T_{0es} = 576.9\ \text{K}$$

Hence, $\Delta T_{0overall} = \eta_t (T_{01} - T_{0es}) = 0.85(900 - 576.9) = 274.6\,\text{K}$

From the velocity triangles, $C_2 = C_a / \cos\alpha_2 = 150 / \cos 75 = 579.5\,\text{m/s}$

$$T_{02} = T_2 + C_2^2 / 2c_p \rightarrow T_2 = T_{02} - C_2^2 / 2c_p$$

Since, there is no change in stagnation temperature in the nozzle,

$$T_2 = T_{01} - C_2^2 / 2c_p = 900 - 579.5^2 / 2 \times 1148 = 753.7\ \text{K}$$

Since this is an impulse turbine, the degree of reaction, $R_x = 0$

$$R_x = \frac{h_2 - h_3}{h_{01} - h_{03}} \quad \text{or} \quad h_2 = h_3 \rightarrow T_2 = T_3 = 753.7\ \text{K}$$

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Solution: Problem # 4

From the velocity triangles at rotor entry,

$$\tan\beta_2 = (C_2 \sin\alpha_2 - U)/C_a = (579.5\sin 75 - 250)/150 = 2.065$$

$$\therefore \beta_2 = 64.16^o$$

$$V_2 = C_a/\cos\beta_2 = 344.15\, m/s$$

We can see that $V_2 = V_3 = C_3$ for constant axial velocity.

Therefore, $T_{03} = T_2 + C_3^2/2c_p = 753.7 + 344.14^2/2\times 1148$

$$= 805.28\ K$$

The temperature drop per stage is

$$T_{01} - T_{03} = 900 - 805.28 = 94.7\ K$$

The number of stages required for the turbine is

$$\Delta T_{0overall}/(T_{01} - T_{03}) = 274.6/94.7 = 2.89 \cong 3 \text{ stages.}$$

Exercise Problem # 1

An axial flow turbine operating with an overall stagnation pressure of 8 to 1 has a polytropic efficiency of 0.85. Determine the total-to-total efficiency of the turbine. If the exhaust Mach number of the turbine is 0.3, determine the total-to-static efficiency. If, in addition, the exhaust velocity of the turbine is 160 m/s, determine the inlet total temperature.

Ans: 88%, 86.17%, 1170.6 K

Exercise Problem # 2

The mean blade radii of the rotor of a mixed flow turbine are 0.3 m at inlet and 0.1 m at outlet. The rotor rotates at 20,000 rev/min and the turbine is required to produce 430kW. The flow velocity at nozzle exit is 700 m/s and the flow direction is at 70° to the meridional plane. Determine the absolute and relative flow angles and the absolute exit velocity if the gas flow is 1 kg/s and the velocity of the through-flow is constant through the rotor.

Ans: α_2=70 deg, β2=7.02 deg, α_3=18.4 deg, β_3=50.37 deg

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Exercise Problem # 3

An axial flow gas turbine stage develops 3.36MW at a mass flow rate of 27.2 kg/s. At the stage entry the stagnation pressure and temperature are 772 kPa and 727°C, respectively. The static pressure at exit from the nozzle is 482 kPa and the corresponding absolute flow direction is 72° to the axial direction. Assuming the axial velocity is constant across the stage and the gas enters and leaves the stage without any absolute swirl velocity, determine (a) the nozzle exit velocity; (b) the blade speed; (c) the total-to-static efficiency; (d) the stage reaction.

Ans: 488m/s, 266.1 m/s, 0.83, 0.128

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Exercise Problem # 4

A single stage axial turbine has a mean radius of 30 cm and a blade height at the stator inlet of 6 cm. The gases enter the turbine stage at 1900 kPa and 1200 K and the absolute velocity leaving the stator is 600 m/s and inclined at an angle of 65 deg to the axial direction. The relative angles at the inlet and outlet of the rotor are 25 deg and 60 deg respectively. If the stage efficiency is 0.88, calculate (a) the rotor rotational speed, (b) stage pressure ratio (c) flow coefficient (d) degree of reaction and (e) the power delivered by the turbine.

Ans: 13550 rpm, 2.346, 0.6, 0.41, 34.6 MW

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