TURBOMACHINERY AERODYNAMICS

Lect- 5

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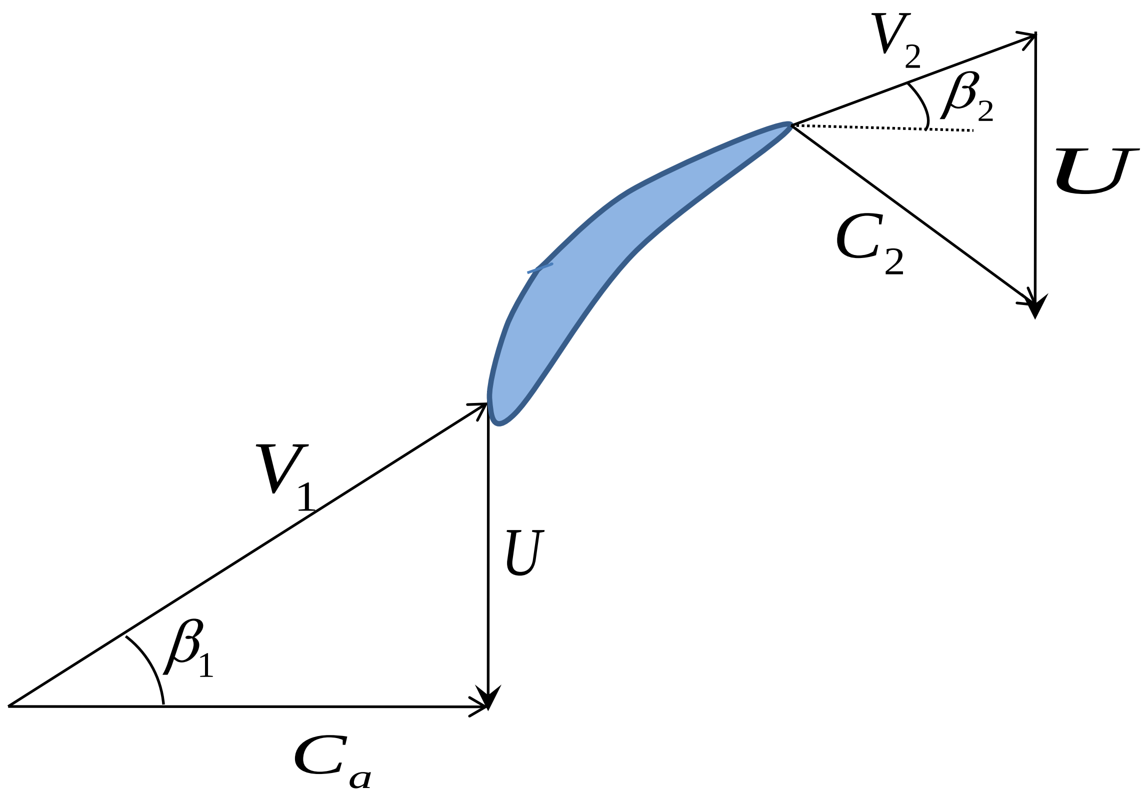
In this lecture...

- Tutorial on 2-D analysis of axial compressors and cascades

Problem 1

Air at 1.0 bar and 288 K enters an axial flow compressor with an axial velocity of 150 m/s. There are no inlet guide vanes. The rotor stage has a tip diameter of 60 cm and a hub diameter of 50 cm and rotates at 100rps. The air enters the rotor and leaves the stator in the axial direction with no change in velocity or radius. The air is turned through 30.2 degree as it passes through the rotor. Assume a stage pressure ratio of 1.2 and overall pressure ratio of 6. Find a) the mass flow rate of air, b) the power required to drive the compressor, c) the degree of reaction at the mean diameter, d) the number of compressor stages required if the isentropic efficiency is 0.85.

Solution : Problem # 1



Solution: Problem # 1

$$U = \pi \times \left(\frac{d_t + d_h}{2}\right) \times N = \pi \times \left(\frac{0.6 + 0.5}{2}\right) \times 100 = 172.76 \text{m / s}$$

$$\beta_1 = \tan^{-1}\left(\frac{U}{C_a}\right) = 49.2°$$

$$\beta_2 = 49.2 - 30.2 = 19°$$

$$\tan\alpha_2 = \left(\frac{U - C_a \tan\beta_2}{C_a}\right) = 80.75$$

$$\alpha_2 = 38.92°$$

Solution: Problem # 1

$$\dot{m} = \frac{\pi}{4} \times \left(d_t^{\,2} - d_h^{\,2}\right) \times C_a \times \rho_2 \quad \& \quad T_1 = T_{01} - \frac{C_a^2}{2C_p} = 276.8\,K$$

$$T_{02} = T_{01} \times \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad T_{02} = 303.41K$$

$$T_2 = 303.41 - \frac{C_2^{\,2}}{2C_p} \quad \& \quad \cos\alpha_2 = \frac{C_a}{C_2}$$

$$\therefore C_2 = \frac{C_a}{\cos\alpha_2} = \frac{150}{\cos 38.92} = 192.79 m/s$$

Solution: Problem # 1

$$T_2 = 303.41 - \frac{192.79^2}{2010} = 284.91\,K$$

$$P_2 = 1.216\,bar$$

$$\rho_2 = \frac{1.216 \times 101325}{287 \times 284.9} = 1.507\,Kg/m^3$$

$$\dot{m} = 19.53\,Kg/s$$

$$P = U \times C_a \times \dot{m} \times (\tan\beta_1 - \tan\beta_2)$$

$$= 172.76 \times 150 \times 19.53 \times (\tan 49.2 - \tan 19) = 412\,KW$$

Solution: Problem # 1

$$R_x = 1 - \frac{C_a}{2U} \times (\tan\beta_1 + \tan\beta_2)$$

$$= 1 - \frac{150}{2 \times 172.76} \times (\tan 49.2 + \tan 19) = 1 - 0.65$$

$$= 0.35$$

$$\Delta T_{0s} = \frac{U \times C_a}{C_p} \times (\tan\beta_1 - \tan\beta_2)$$

$$= \frac{172.76 \times 150}{1005} \times (\tan 49.2 - \tan 19) = 20.99\,K$$

Solution: Problem # 1

$$\Delta T_{0overall} = \frac{T_1}{\eta_C} \times \left(\pi_C^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$= \frac{288}{0.85} \times \left(6^{0.286} - 1\right) = 226.5K$$

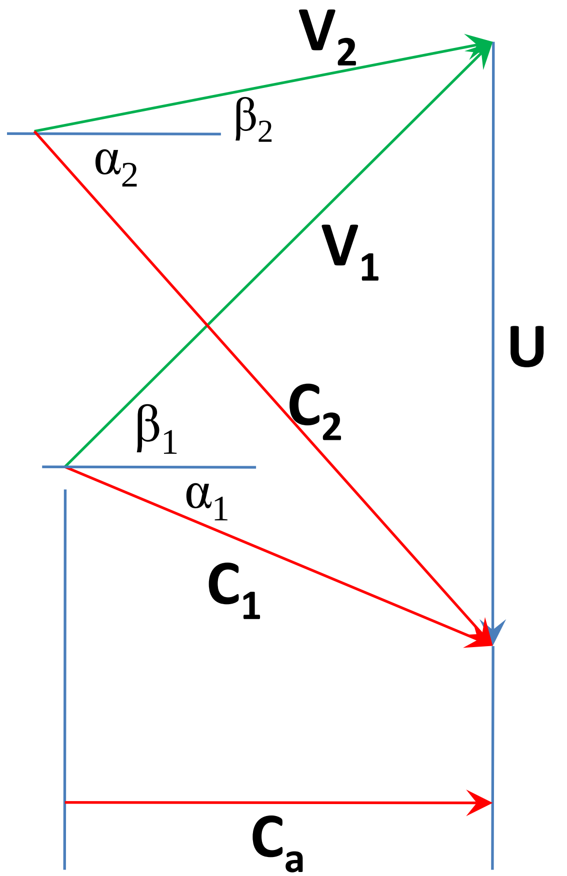
$$n = \frac{226.5}{20.99} = 10.79 \approx 11$$

Therefore the number of stages required for the given pressure ratio is 11.0.

Problem # 2

An axial flow compressor is to be designed to generate a total pressure ratio of 4.0 with an overall isentropic efficiency of 0.85. The inlet and outlet blade angles of the rotor blades are 45 degree & 10 degree respectively and the compressor stage has a degree of reaction of 50 percent. If the blade speed is 220 m/s and the work done factor is 0.86, find the number of stages required. Is it likely that the compressor will suffer from shock losses? The ambient air static temperature is 290 K and the air enters the compressor through guide vanes.

Solution: Problem # 2



Solution: Problem # 2

Axial velocity, $C_a = \dfrac{U}{\tan\beta_1 + \tan\beta_2} = 187 m/s$

Absolute velocity at inlet, $C_1 = \dfrac{C_a}{\cos\alpha_1} = 190 m/s$

The per stage temperature rise,

$$\Delta T_{0s} = \frac{\lambda \times U \times C_a \times (\tan\beta_1 - \tan\beta_2)}{C_p} = 29K$$

Solution: Problem # 2

Total temperature at compressor inlet,

$$T_{02} = T_2 + \frac{C_1^2}{2Cp} = 331.8K$$

Isentropic total temperature at compressor exit,

$$T_{03s} = T_{02} \times \pi_c^{\frac{\gamma-1}{\gamma}} = 493.9K$$

Actual total temperature at compressor exit,

$$T_{03} = T_{02} + \frac{(T_{03s} - T_{02})}{\eta_c} = 522.5K$$

Solution: Problem # 2

Therefore total temperature rise across the compressor = $T_{03} - T_{02} = 190.74K$

The number of stages required=

$$\frac{\text{Overall temperature rise across the compressor}}{\text{Per stage temperature rise}}$$

$$= \frac{190.74}{29} = 6.6 \approx 7$$

Solution: Problem # 2

To determine whether the compressor will suffer from shock losses, we need to find the relative Mach number

$$M_{rel} = \frac{V_1}{\sqrt{\gamma R T_2}}$$

$$V_1 = \frac{C_a}{\cos \beta_1} = 264.5 m/s$$

$$\therefore M_{rel} = 0.77$$

Since relative Mach number is less than unity, the compressor is not likely to suffer from shock losses.

Problem # 3

- The conditions of air at the entry of an axial compressor stage are P_1=1 bar and T_1=314 K. The air angles are β_1=51°, β_2=9°, α_1= α_3=7°. The mean diameter and peripheral speed are 50 cm and 100 m/s respectively. Given that the work done factor is 0.95, stage efficiency is 0.88, mechanical efficiency is 0.92 and the mass flow rate is 25 kg/s, Determine a) air angle at stator entry, b) blade height at entry and hub–tip diameter ratio, c)Stage loading coefficient, d) Power required to drive the stage.

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Solution: Problem # 3

a)

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

$$\frac{100}{C_a} = \tan 7 + \tan 51 \qquad \therefore C_a = 73.65 m/s$$

$$\tan\alpha_2 + \tan\beta_2 = \frac{U}{C_a}$$

$$\tan\alpha_2 + \tan 9 = \frac{100}{73.65} \qquad \therefore \alpha_2 = 50.18°$$

Solution: Problem # 3

b)

$$\dot{m} = \rho \times C_a \times (\pi \times d \times h),$$

Substituting known values in the above, $h = 0.19\,m$

$$d_t = 50 + 19 = 69\,cm,$$

$$d_h = 50 - 19 = 31\,cm$$

The hub - tip ratio is $\dfrac{d_h}{d_t} = 0.449$

Solution: Problem # 3

c) $\Psi = \frac{w}{U^2}$ & $\quad w = \lambda \times C_a \times U \times (\tan\beta_1 - \tan\beta_2)$

$$w = 0.95 \times 100 \times 73.65 \times (\tan 51 - \tan 9) = 7534.8 J/Kg$$

$$\Psi = \frac{7534.8}{100^2} = 0.7535,$$ is the loading coefficient.

d) $$P = \frac{\dot{m} \times w}{\eta_m} = 204.75 KW$$ is the power required.

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