TURBOMACHINERY AERODYNAMICS

Lect- 32

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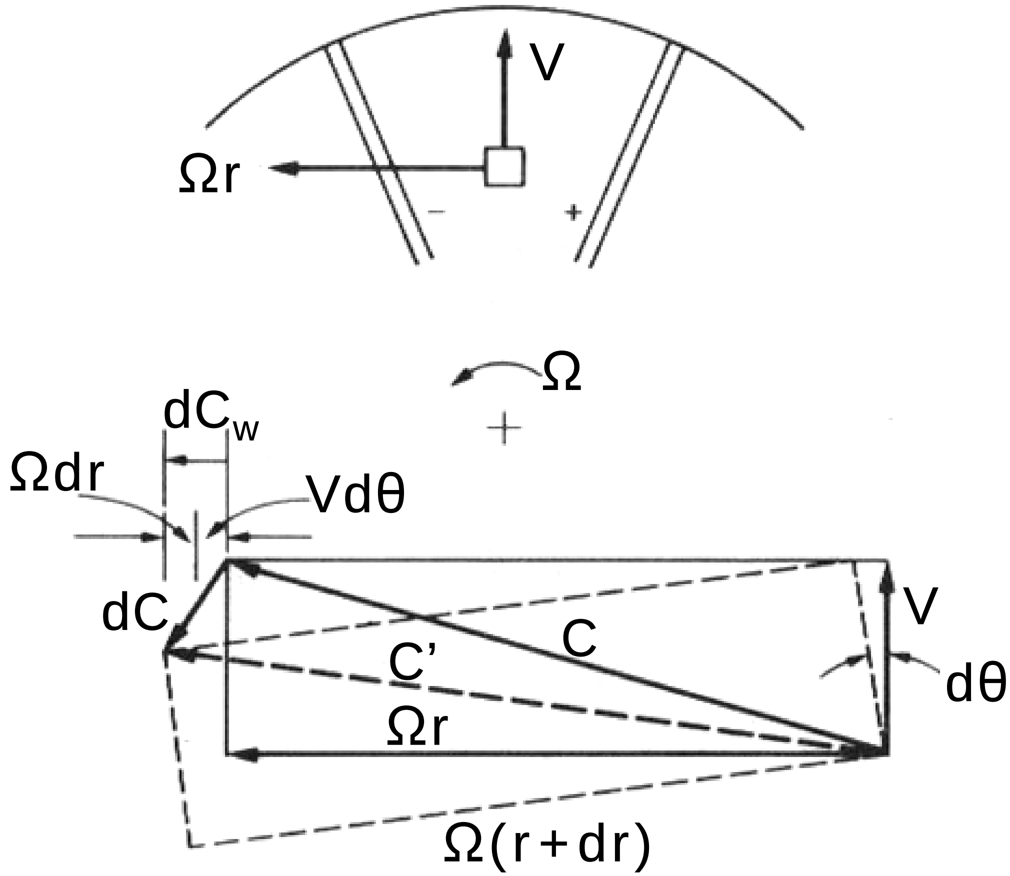
In this lecture...

- Centrifugal compressors
  - Coriolis acceleration
  - Slip factor
  - Performance characteristics
  - Stall and surge

Coriolis acceleration

- We have discussed earlier that pressure change due to the centrifugal force field is not a cause of boundary layer separation.
- This can also be explained by the Coriolis forces that are present in centrifugal compressor rotors.
- Let us consider a fluid element travelling radially outward in the passage of a rotor.
- We shall examine the velocity triangles of this fluid during a time period *dt.*

Coriolis acceleration



Coriolis acceleration

- The magnitude of the relative velocity is unchanged, but the particle has suffered an absolute change of velocity.

$$dC_w = \Omega dr + V d\theta$$

$$\text{or, } dC_w = \Omega V dt + V\Omega dt,$$

Thus, the Coriolis acceleration , $a_\theta = 2\Omega V$

and it requires a pressure gradient in the tangential direction of magnitude, $\frac{1}{r}\frac{\partial P}{\partial \theta} = -2\rho\Omega V$

Coriolis acceleration

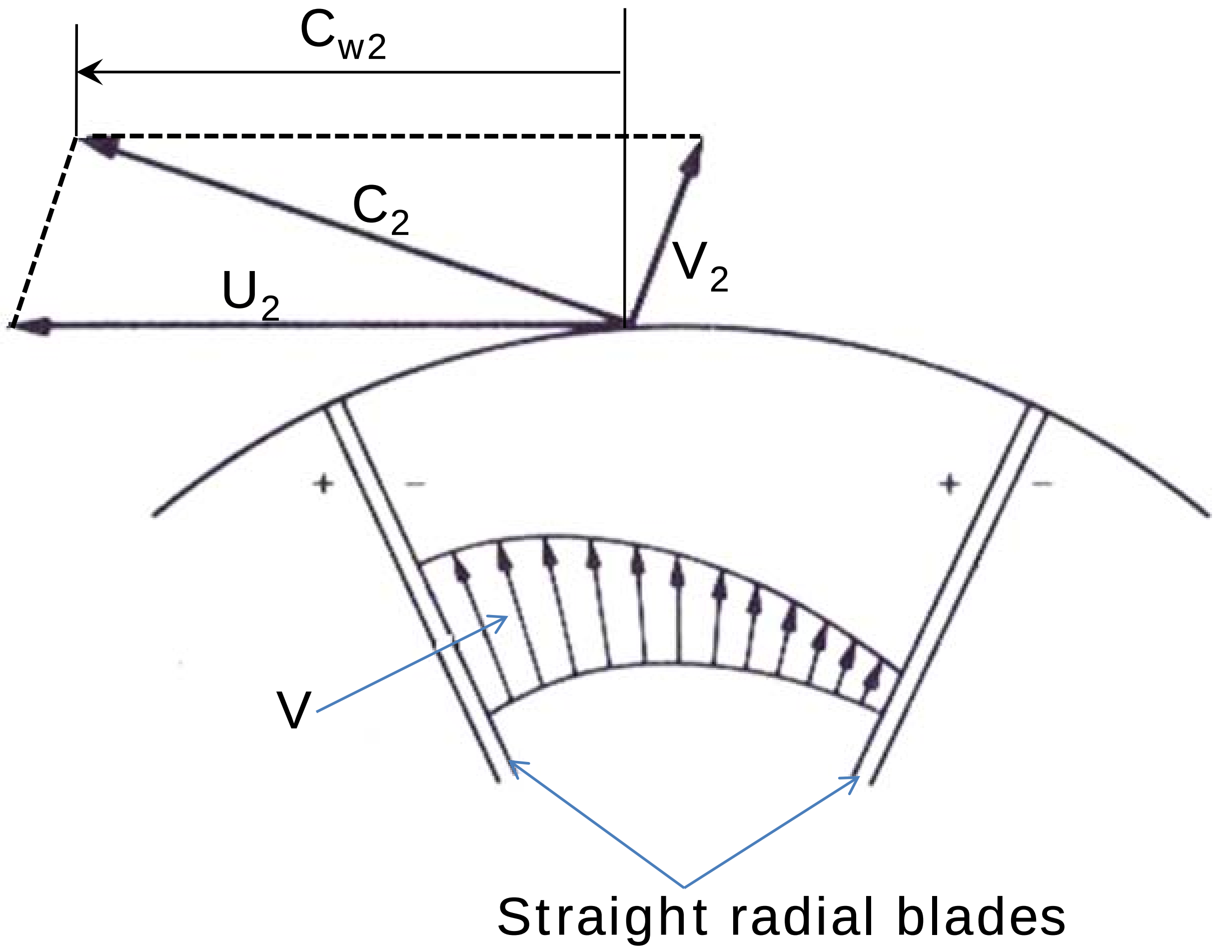
- The existence of the tangential pressure gradient means that there will be a positive gradient of V in the tangential direction.

$$\frac{1}{\rho}\frac{dP}{rd\theta} = -\frac{d\left(V^2/2\right)}{rd\theta} = -\frac{V}{r}\frac{dV}{d\theta}$$

$$\text{Therefore, } \frac{1}{r}\frac{dV}{d\theta} = 2\Omega$$

- This means that there will be a tangential variation in relative velocity.

Coriolis acceleration



Slip factor

- Towards the outlet of the impeller, as the Coriolis pressure gradient disappears, there will be a difference between C_{w2} and U_2.
- This difference in the velocities is expressed as slip factor, $\sigma_s = C_{w2} / U_2$
- The slip factor is approximately related to the number of blades of the impeller.
- For a straight radial blade, the slip factor is empirically expressed as $\sigma_s \approx 1 - 2/N$, where N is the number of blades.

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Slip factor

- As the number of blades increases, the slip factor also increases and thus the slip lag at the tip of the impeller reduces.
- The effect of slip is to reduce the magnitude of swirl velocity and therefore the pressure ratio.
- The presence of slip means that to deliver the same pressure ratio, either the impeller diameter or the rotational must be increased.
- This in turn may lead to either increase in frictional losses or stresses on the impeller.

Performance characteristics

- The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.
- Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.
- Centrifugal compressors also suffer from instability problems like surge and rotating stall.

Performance characteristics

- The compressor outlet pressure, P_{02}, and the isentropic efficiency, $\eta_{C,}$ depend upon several physical variables

$$P_{02}, \eta_C = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non - dimensionless parameters,

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\gamma R T_{01}}}{P_{01} D^2}, \frac{\Omega D}{\sqrt{\gamma R T_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

The above reduces to $\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$

Performance characteristics

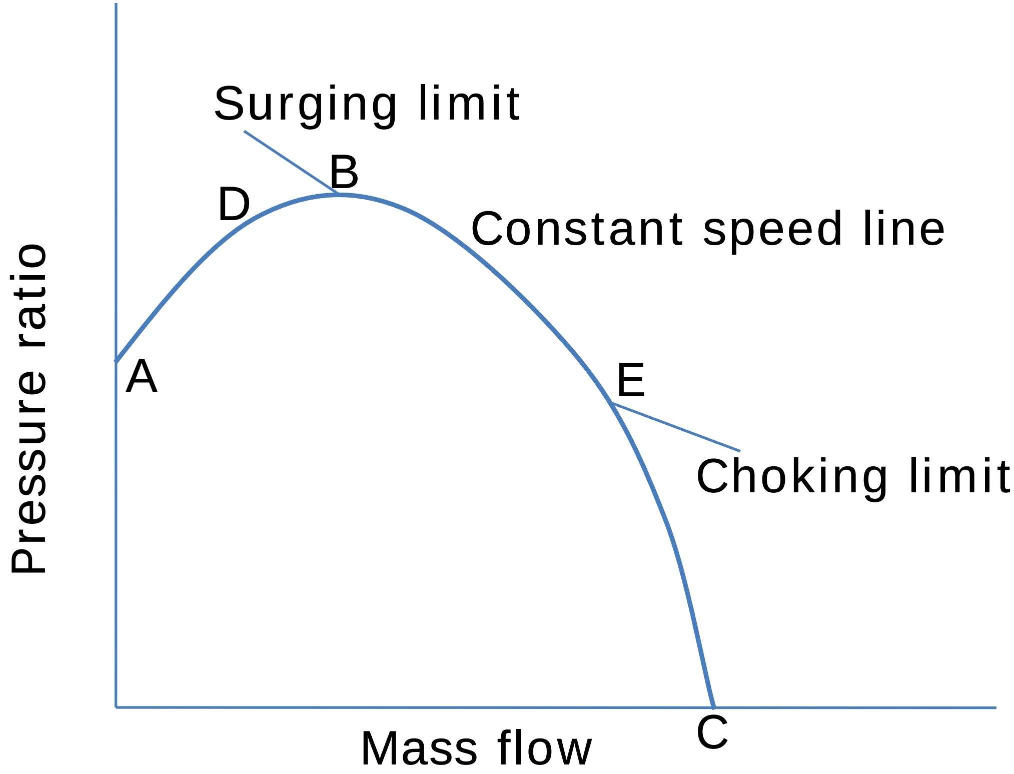
Usually, this is further processed in terms of the standard day pressure and temperature.

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)$$

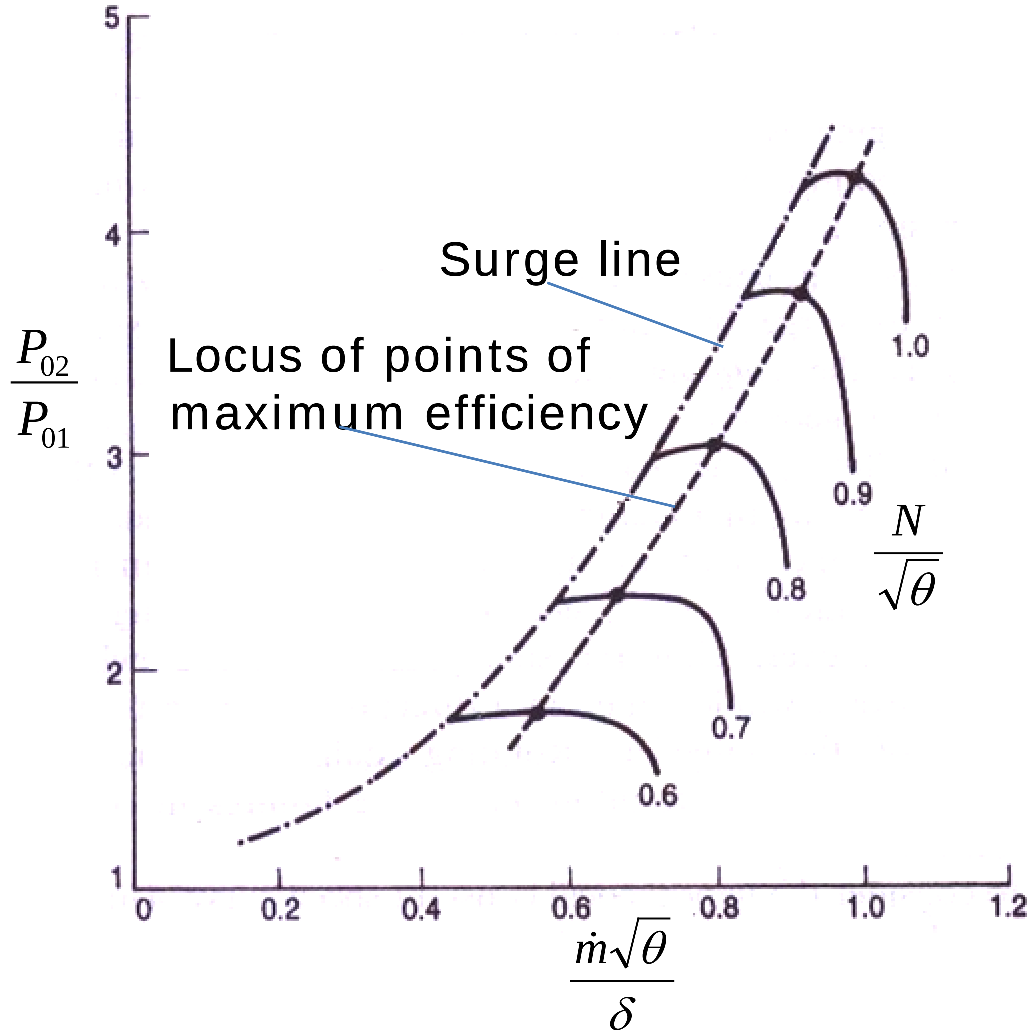
Where, $\theta = \dfrac{T_{01}}{\left(T_{01}\right)_{Std.\,day}}$ and $\delta = \dfrac{P_{01}}{\left(P_{01}\right)_{Std.\,day}}$

$$\left(T_{01}\right)_{Std.\,day} = 288.15\,K \text{ and } \left(P_{01}\right)_{Std.\,day} = 101.325\text{ kPa}$$

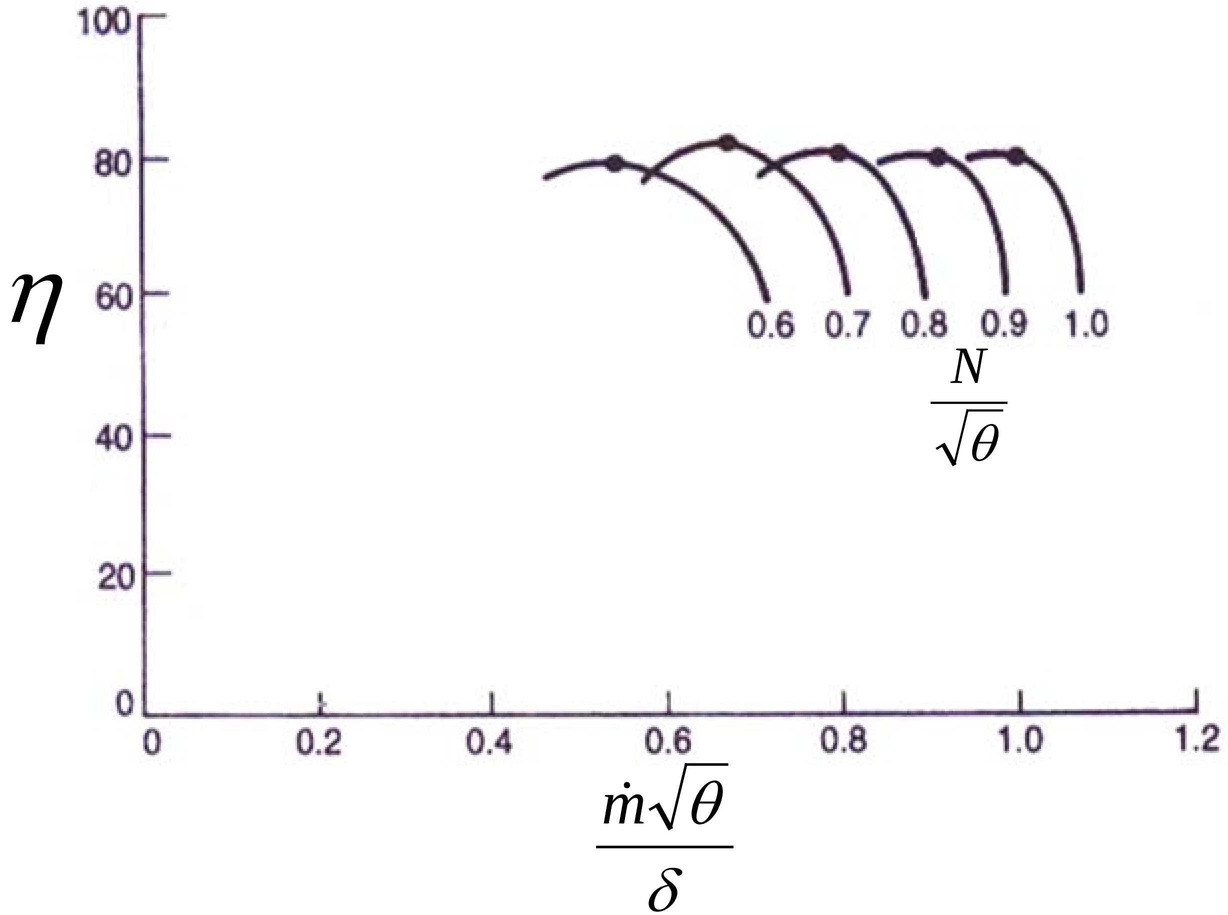
Performance characteristics



Performance characteristics



Performance characteristics



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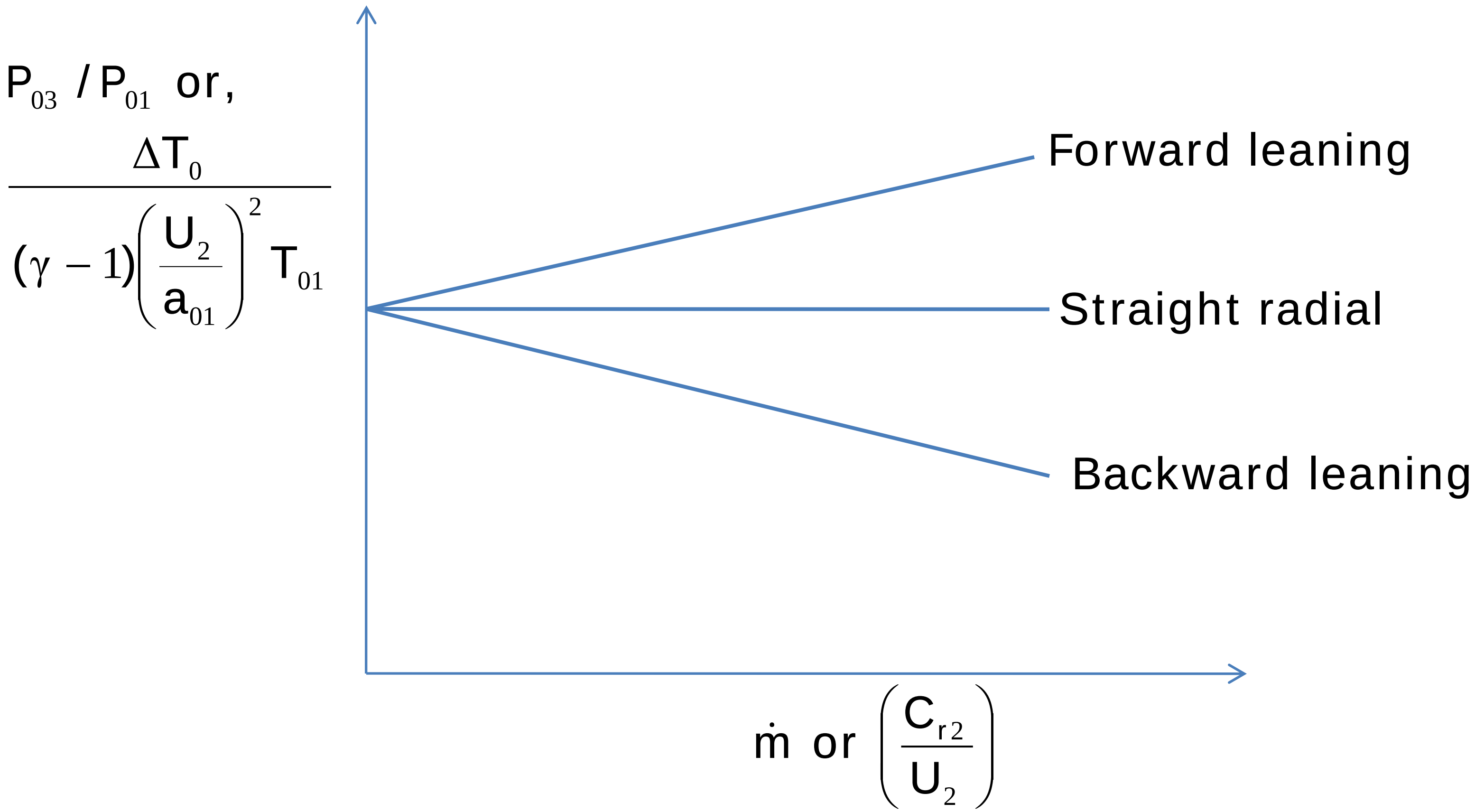
Performance characteristics

- There are two limits to the operation of the compressor.
- Operation between A and B are limited due to occurrence of surge.
- Surging: sudden drop in delivery pressure and violent aerodynamic pulsations.
- Operation on the positive slope of the performance characteristics: unstable
- Surging usually starts to occur in the diffuser passages.

Performance characteristics

- The pressure ratio or the temperature rise in a centrifugal compressor also depends upon the blade shaping.
- There are three possible types of blade shapes: forward leaning, straight radial and backward leaning.
- Theoretically, the forward leaning blading produces higher pressure ratio for a given flow coefficient.
- However such a blading has inherent dynamic instability.
- Therefore, straight radial or backward leaning blades are popularly used.

Performance characteristics



Performance characteristics for different blade geometries

Rotating stall

- Rotating stall might also affect the compressor performance.
- In this case a stall cell (that might cover one or more adjacent blades) rotates within the annulus.
- Full annulus rotating stall may eventually lead to surge.
- Rotating stall may also lead to aerodynamically induced vibrations and fatigue failure of the compressor components.

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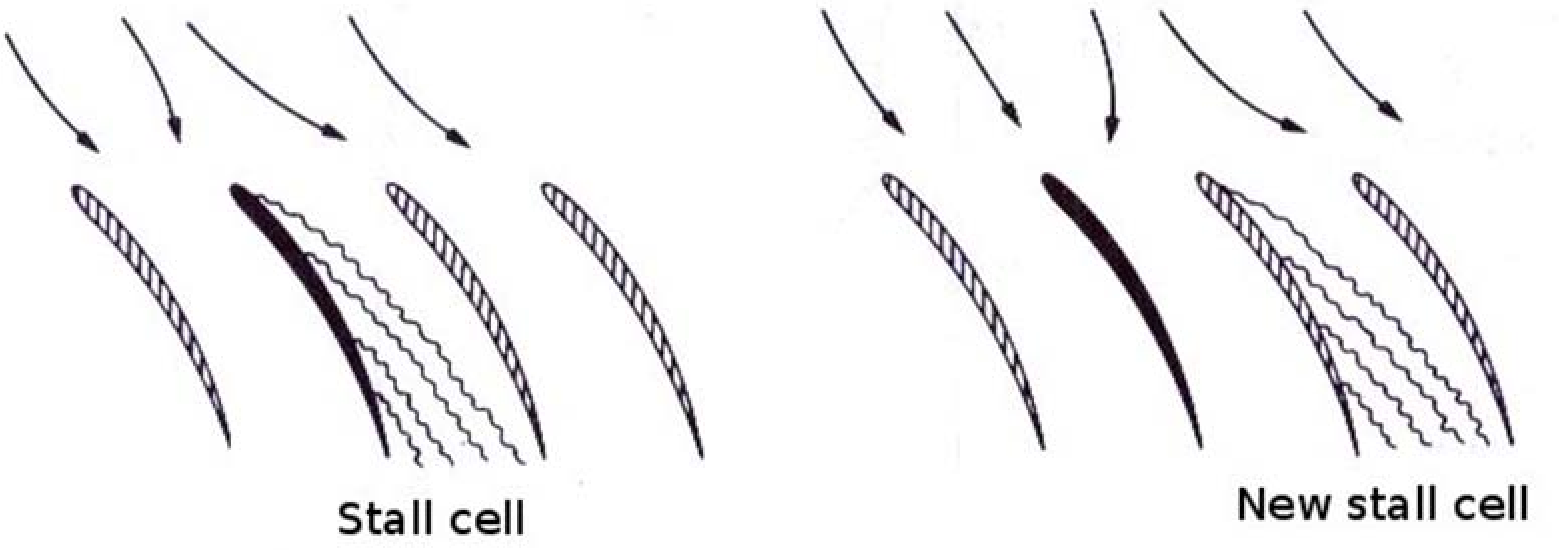
Rotating stall



Propagation of rotating stall

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Choking in a compressor stage

- The other limiting aspect of centrifugal compressors is choking.
- As the mass flow increases, the pressure decreases, density reduces.
- After a certain point, no further increase in mass flow will be possible.
- The compressor is then said to have choked.
- The right hand side of the constant speed lines together form the choking line.

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Choking in a compressor stage

- Choking behaviour for rotating passages is different from that of stationary passages.
- Inlet:
  - Choking takes place when M=1

$$\frac{T}{T_0} = \frac{2}{\gamma + 1}$$

Assuming an isentropic flow, the choking mass flow rate is

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)}$$

  - Since ρ_0, a_0 refer to the inlet stagnation conditions and are constant, the mass flow rate is also a constant: choking mass flow.

Choking in a compressor stage

- Impeller:
  - In rotating passages, the flow conditions are referred through rothalpy, I.
  - During choking, it is the relative velocity, V, that becomes equal to the speed of sound.

$$I = h + \tfrac{1}{2}(V^2 - U^2) \rightarrow T_{01} = T + (\gamma RT / 2c_p) - (U^2 / 2c_p)$$

$$\therefore \frac{T}{T_{01}} = \left(\frac{2}{\gamma+1}\right)\left(1 + \frac{U^2}{2c_p T_{01}}\right) \text{ and } \frac{\dot{m}}{A} = \rho_{01} a_{01} \left(\frac{T}{T_{01}}\right)^{(\gamma+1)/2(\gamma-1)}$$

$$\text{or, } \frac{\dot{m}}{A} = \rho_{01} a_{01} \left[\frac{2 + (\gamma-1)U^2 / a_{01}^2}{\gamma+1}\right]^{(\gamma+1)/2(\gamma-1)}$$

Choking in a compressor stage

- In an impeller, the choking mass flow is a function of the rotational speed.
- Therefore, the compressor can, in principle, handle a higher mass flow with an increase in speed.
- This also requires that no other component like the inlet or the diffuser undergoes choking at this new rotational speed.

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Choking in a compressor stage

- Diffuser:
  - The choking mass flow in a diffuser has an equation similar to that of an inlet:

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}$$

  - The stagnation conditions at the inlet of diffuser depend upon the impeller exit conditions.
  - It can be shown that the choking mass flow is a function of the rotational speed and therefore can be varied by changing the rotational speed.

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In the next lecture...

- Tutorial on centrifugal compressors