



# TURBOMACHINERY AERODYNAMICS

Lect - 30

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Axial Flow Turbine

3-D blade design

**Design Steps** : Design steps in brief are

- Selection of parameters
- Design at mean diameter,  $D_m$
- Radial variation of parameters
- Profiling of stator and rotor blades

## 1. Design at Dm

a) Parameters to be selected from the preliminary cycle (thermodynamic) calculations are: (@ flight velocity & altitude),

$T_g, \dot{m}_g, P_a, T_a$  at design point  $P_g^*, \pi, \tau, P_T^*$

and the engine parameters

$$\eta_T^* = 0.88 - 0.90 \text{ for one stage}$$

$$= 0.91 - 0.94 \text{ for each stage for multi-stage}$$

b) Parameters selected from compressor-turbine matching are:

Exit  $U_{mean}, D_{mean}, D_{tip}, \bar{d}, \lambda_2$  or  $M_2$

[ $\lambda$  is a total temperature based critical speed ratio,  $\lambda = v/a_{cr}$  ]

$$\text{and, } a_{cr} = \sqrt{\gamma_g RT_{0i}}$$

All these should lead to a work distribution,  $H_T = \sum_1^z H_i$   
and number of stages Z.

## 2. Design at $D_m$

Matching : Peripheral velocity,  $u$  is selected from turbine-compressor matching criteria. The outlet velocity from the nozzle may be supersonic, but inlet relative velocity to rotor is generally brought down below sonic speeds. The respective sonic speeds are:

At nozzle exit

$$a_2 = \sqrt{\gamma R T_2}$$

At rotor exit

$$a_3 = \sqrt{\gamma R T_3}$$

Total Temp based critical speed at the nozzle exit

$$a_{cr2} = \sqrt{\gamma_g R \cdot T_{02}}$$

Blade passage wall temperature at  $Dm$

$$T_{wt} = T_{0t} - \frac{C_2^2 - V_2^2}{2C_p \gamma R} = T_{0t} - \frac{\gamma - 1}{2} (C_2^2 - V_2^2)$$

$$V_2^2 = C_2^2 + U^2 - 2UC_2 \sin \alpha_2$$

$$\frac{T_{wt}}{T_{0t} + 1} = 1 - \frac{\gamma - 1}{\lambda \cdot \lambda} \cdot \left( \sin^2 \alpha_u - \lambda^2 \right)$$

and

$$\frac{\lambda_{2-abs}}{\lambda_{2-rel}} = \frac{\cos \beta_2}{\cos \alpha_2} \sqrt{\frac{T_{wt}}{T_{0t}}}$$

where

$$\lambda_{2-abs} = C_2 / a_{cr}$$

$$\lambda_{2-rel} = V_2 / a_{cr}$$

$$\lambda_u = U / a_{cr}$$

Choice of velocity triangle for the rotor inlet is obtained by the relation,

$$M_{2-rel} = J_1 (M_2 - M_{2u}), \quad \text{where}$$

$$J_1 = \sqrt{1 + \frac{1 - \sin \alpha_1}{\frac{M_1}{M_{1u}} + \frac{M_{1u}}{M_1} - 1}}$$

**= 1.2 to 1.4, is a flow coefficient**

At the exit of the stage, Mach number can be expressed as,

$$M_3 = J_2 \left( M_{3-rel} - M_{3u} \right)$$

Where, another flow coefficient  $J_2$  is used,

$$J_2 = \sqrt{1 + \frac{1 - \sin\beta_3}{\frac{M_{3-rel}}{M_{3u}} + \frac{M_{3u}}{M_{3-rel}} - 1}} = 2.5 \text{ to } 4.5$$

$$M_{3u} = M_{2u} \cdot \sqrt{\frac{T_2}{T_3}}$$



For moderate pressure drops ( $\pi < 2.0$ ) the flow in the rotor may be entirely subsonic. However for high pressure drops ( $\pi > 2.5$ ), the flow becomes transonic at the stator trailing edge.

**Stator Exit flow conditions:**

**From continuity, for an unit length of the blade, at throat**

$$\rho_t \cdot V_t \cdot s_t \cdot l = s \cdot l \cdot \rho_3 \cdot V_3 \cdot \sin \beta_{3eff}$$

*s = blade spacing or pitch*

*s<sub>t</sub> = O = Throat width,*

Subscript **t** for throat

*β<sub>3</sub>' = blade exit angle*

*β<sub>3</sub> = flow exit angle*

*β<sub>3-eff</sub> = Mass - averaged effective flow exit angle*

Generally, because of diffusion

$$\rho_t \cdot V_t > \rho_3 \cdot V_3$$

Subscript *t* for throat

$$\sin \beta_{3-eff} = \frac{s_t}{s} \quad \text{Throat area ratio}$$

The exact relationship between  $\beta_2$  and  $s_t / s$  can be found experimentally by accurate cascade analysis

$$\beta_3 = \sin^{-1} \left( k_2 \frac{s_t}{s} \right) \quad \text{Initially assume } K_2 = 1$$

a) Actual  $R_x$  ,

$$R_x = \frac{h_{rotor}}{h_{rotor} + h_{stator}} = \frac{h_{rotor}}{h_T} = \frac{V_3^2 - V_2^2}{2h_T}$$

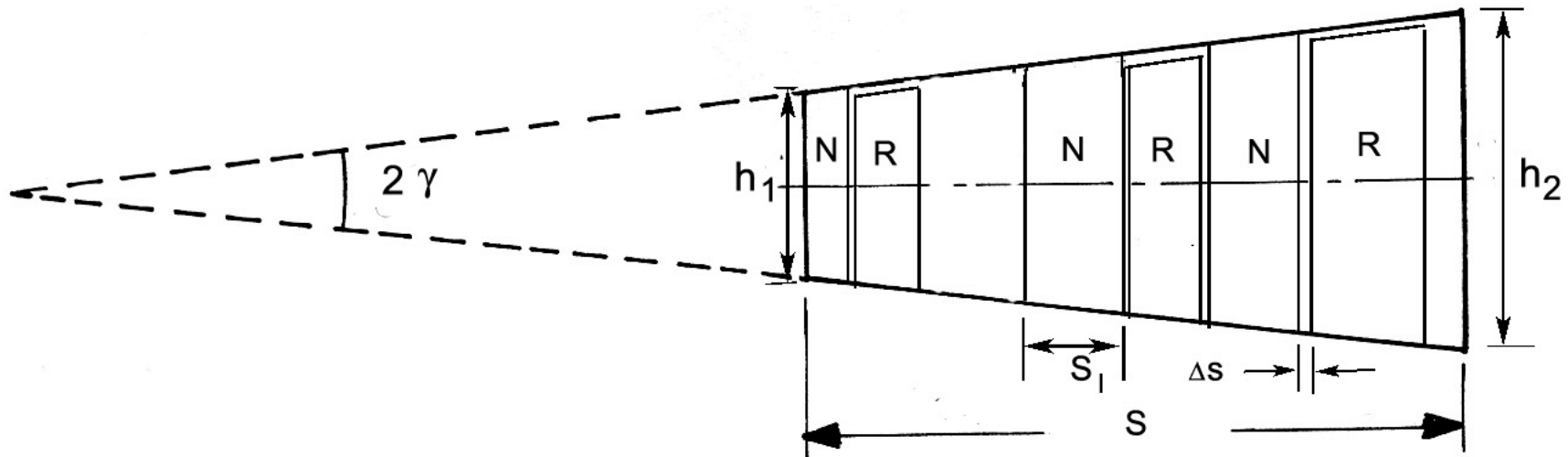
Degree of Reaction - ideal to actual change

$DR$	0	0.1	0.25	0.35	0.45	0.5
$DR_{act}$	0.03	0.073	0.226	0.33	0.433	0.485

## b) Distribution of work in a multi-stage turbine

$$H_T = \sum_1^z H_i$$

## c) Selection of flow track i.e. $\lambda$ angle (local)



d) Velocity triangle and other parameters.

i) 
$$H_T = \frac{H_{T-actual}}{\overline{W}_{rotor}}, \text{ where } \overline{W}_{rotor} = 0.97 - 0.98$$
 (Loss coefficient)

ii) 
$$C_{2w} + C_{3w} = \frac{H_T}{U}$$

$$C_{2w} - C_{3w} = 2U(1 - DR)$$

From which  $C_w$  are selected at mean diameter

- iii) Select three probable values of  $\alpha_2$  and for each one of them calculate the velocity triangle parameters at various radial stations

$$C_{2a} = \frac{C_{2w}}{\tan \alpha_2} \quad \text{where, } C_{\overline{2}} = \frac{C_{2a}}{\cos \alpha_2}, \quad 2C_{\overline{2}} = \frac{C_2}{a_{cr2}}$$

And,  $\lambda_{cr2} = 0.85 \text{ to } 0.9$

- iv) Assume velocity coefficient  $\phi$ , and calculate pressure loss coefficient  $\delta_{noz}$

$$\delta_{noz} = \frac{\pi \left( \frac{\lambda_2}{\phi} \right)}{\pi (\lambda_2)}; \quad P_{02} = P_{01} \cdot \delta_{noz}$$

v) 
$$A_2 = \frac{\dot{m}_g}{C_2 a}$$

From which  
Blade height 
$$h_{2_{bl}} = \frac{A_2}{\pi D_{m2}}$$

$$h_{2_{passage}} \Delta r = h_{2_{bl}} \left( 1 + \bar{\Delta r} \right)$$

the rotor tip  
radial gap 
$$\bar{\Delta r} = \frac{\Delta r}{h_{1_{bl}}} = 0.010 - 0.015$$

$$\text{vi) } \bar{d} = \frac{\left( D_{m2} / h_{2bl} \right) - 1}{\left( D_{m2} / h_{2bl} \right) + 1}$$

$$\text{vii) } \beta_2 = \tan^{-1} \left( \frac{C_{2w} - U}{C_{2a}} \right)$$

Find  $V_2$  and  $\lambda_{2-rel}$



viii)  $C_{2a}$ ,  $C_2$ , and  $V_2$ , leads to  $\lambda_3$ ,  $\alpha_3$  and  $\beta_3$ . Calculate for blade geometry and check for  $\lambda_2, \alpha_2$  within limits mentioned.

$$\text{also } \lambda_{2-rel} \leq 1.0 - 1.1$$

ix) Assume  $\psi$  for rotor, calculate  $P_{02-rel}$  and calculate  $P_{03-rel}$

From empirical rotor loss correlations, e.g. those given here:

$$P_3^* = P_{3-rel}^* \frac{\pi(\lambda_{3-rel})}{\pi(\lambda_3)}; \quad P_3 = P_3^* \cdot (3) \quad \pi_T^* = \frac{P_{01}^*}{P_{03}^*}$$

The stage efficiency is calculated from :

$$\eta_{T_i} = \frac{H_{T_i}}{\frac{k_g}{k_g + 1} R_g T_{g_i}^* \left( 1 - \frac{1}{\pi_T^{*\frac{k_g-1}{k_g}}} \right)}$$

The best efficiency consideration often determines the selection of  $\alpha_2$  from the three initial considered. In some cases e.g. military a/c engine, best pressure ratio  $\pi_{0T}$  may be used for making the final decision on  $\alpha_2$

Exit area  $A_3$  may now be found from various aerothermodynamic parameters and using the continuity condition.

$$h_{3_{bl}} = \frac{D}{2\pi} \sqrt{\frac{D^2}{4} - \frac{A_3}{\pi}}$$

For  $D_{tip} = \text{const}$

$$d = D_m - h_{bl}$$

$$h_{3_{bl}} = \sqrt{\frac{d^2}{4\pi} - \frac{A_3}{2\pi}} - \frac{d}{2}$$

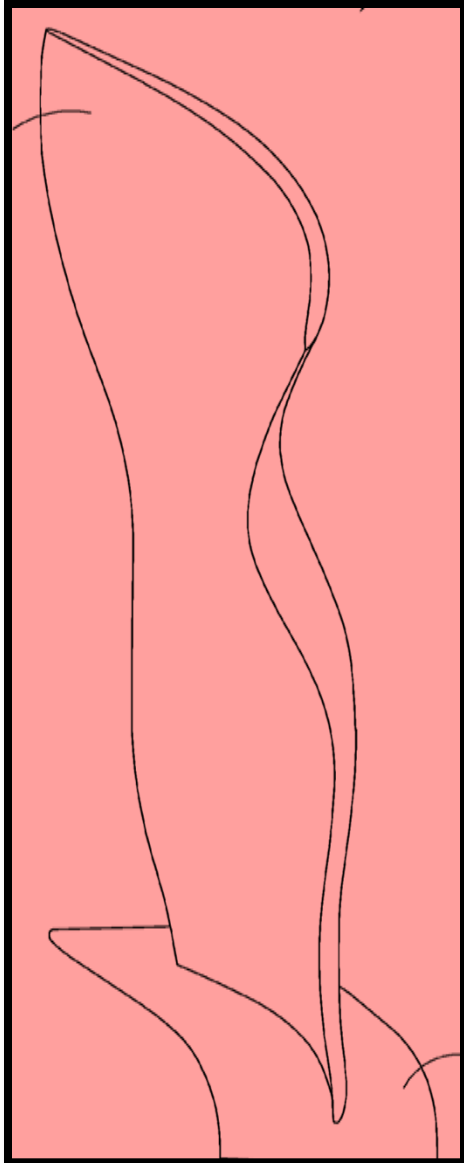
For  $d_{hub} = \text{const}$

$$D = d + 2h_{bl}$$

After these calculations divergence angle is checked and if  $\gamma > 15^\circ$ , the blades angles are modified to allow for more expansion.

Radial variation: either use  $\alpha_2$  as constant from hub to tip or use some vortex law e.g. constant reaction law or the free vortex law

Profiling: same as in last lect use turbine specific airfoils e.g. T6 (HPT) or T106 (LPT) airfoils



A modern turbine blade obtained through design, optimization and stress calculations