# TURBOMACHINERY AERODYNAMICS

Lect 25

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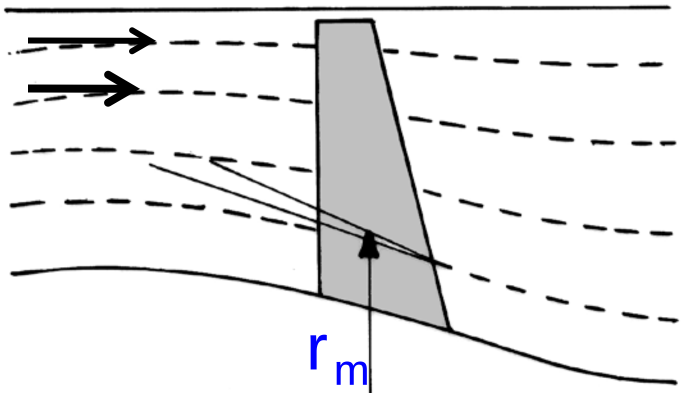
# Axial Flow Turbines

# 3-D Flow theories

- It is assumed that radial motion takes place in the blade passage only

$$C_r << C_a \quad ; \quad C_r << C_w$$

- The stream surface has a cylindrical shape




- simplified radial equilibrium equation is valid

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}.C_w^2$$

Following Three 3-D flow models in axial turbines are often used for design and analysis

1) Free Vortex flow

2) Constant nozzle exit angle, $\alpha_2$

3) Arbitrary vortex case, $C_w = r^n$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## 1) Free Vortex Flow model

$C_w \, . \, r = \text{constant}$, applied on the rotor flow which normally entails a few assumptions :

At turbine rotor entry ,

$dH_{02}/dr = 0$ ; $C_{w2}.r = \text{constant}$; $C_{a2} = \text{const}$

Rotor specific work done :

$$H_{02} - H_{03} = U\,(C_{w2} + C_{w3}) = \omega(r_2.C_{w2} - r_3.C_{w3}) = \text{constant}$$

With $C_{w3}.r = \text{constant}$, it follows $C_{a3} = \text{const}$


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Hence, for obtaining various parameters along blade length following may be adopted:

1) All thermodynamic properties are constant in the annulus

2) $\tan\alpha_2 = (r_m/r)_2 \tan \alpha_{2m}$

3) $\tan\beta_2 = (r_m/r)_2 \tan \alpha_{2m} - (r/r_m)_2 . U_m/C_{a2}$

4) $C_{w3}.r$ = constant, $C_{a3}$ = const $= C_{a2}$

5) $\tan\alpha_3 = (r_m/r)_3 \tan \alpha_{3m}$

6) $\tan\beta_3 = (r_m/r)_3 \tan \alpha_{3m} + (r/r_m)_3 . U_m/C_{a3}$


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## Constant Nozzle exit angle model

This model has been utilized for the practical purpose of creating stator-nozzle blades with zero twist. When stator-nozzles are facing very high inlet temperature elaborate cooling mechanism is embedded inside the blades; to facilitate efficient cooling of the blades, it is thought that such blades may not be twisted at all.

$$\alpha_2 = \text{constant}$$


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$$\cot \alpha_2 = \frac{C_{a2}}{C_{w2}} = const$$

$$C_{a2} = C_{w2}.\cot \alpha_2; \quad which\ yields\ \frac{dC_{a2}}{dr} = \frac{dC_{w2}}{dr}.\cot \alpha_2$$

Now invoking the *radial equilibrium equation* in energy eqn

$$\frac{dH}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w{}^2}{r} \quad \text{and,} \quad \frac{dH}{dr} = 0$$

We get, $C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w{}^2}{r} = 0$

$$C_{w2}.\cot^2 \text{ȣ}_2.\frac{dC_{w2}}{dr}+C_{w2}.\frac{dC_{w2}}{dr}+\frac{{C_{w2}}^2}{r}=0$$

$$C_{w2}\left(1+\cot^2 \text{ȣ}_2\right).\frac{dC_{w2}}{dr}+\frac{{C_{w2}}^2}{r}=0$$

$$\frac{dC_{w2}}{dr}=-\sin^2 \text{ȣ}_2.\frac{dr}{r}$$

which, on integration yields

$C_{w2}.r^{\sin^2\alpha_2} = const$ ; and then $C_{w2} = C_{w2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$

alternate l y, $C_{a2}.r^{\sin^2\alpha_2} = const$

and then $C_{a2} = C_{a2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$

and finally in terms of absolute velocity,

$$C_2 = C_{2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$$



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So, at the rotor inlet station one can say,

if $\alpha_2$ = constant

then,

$$\frac{C_{w2}}{C_{w2m}} = \frac{C_{a2}}{C_{a2m}} = \frac{C_2}{C_{2m}} = \frac{r}{r_m}$$



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Now,
There are three possibilities:

a) Constant $H_{03}$ at the rotor outlet

b) Zero whirl velocity at the outlet, i.e. $\alpha_3 = 0$

c) Free Vortex continued at the outlet


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a) Constant Total Enthalpy at the outlet condition, if applied

$$U\,(C_{w2} + C_{w3}) = \Delta H_0$$

And, whirl component of the velocity at rotor outlet is found from :

$$C_{w3} = \frac{\Delta H_0}{U} - C_{w2} = \frac{K}{r} - C_{a2}\tan\alpha_2$$

$$\text{where, } K = \Delta H_0 / \omega$$

And, subsequently $C_{a3}$ may be also computed Both of which are computed from root to tip, using the variation shown in slide 10


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b) Zero rotor exit whirl velocity $\alpha_3$ = 0

this means, $dH/dr = C_{a3} . dC_{a3}/dr$

And ,

$H_{03} = H_{02} - U\, C_{w2} = H_{02} - U.C_{w2m} (r_m/r)^{\sin^2\alpha}$

Which, produces the enthalpy distribution radially at exit :

$$\frac{dH_{03}}{dr} = \frac{d}{dr}\left[ U.C_{w2m} \cdot \left(\frac{r_m}{r}\right)^{\sin^2\alpha_2} \right]$$


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c) Free Vortex continued at rotor exit

The exit axial velocity field may be expressed as :

$$C_{a3}^2 = C_{a3m}^2 + 2U_m C_{w2m}\left[1 - \left(\frac{r}{r_m}\right)^{\cos^2\alpha}\right]$$


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## 3) Arbitrary vortex case, $C_w = r^n$

Depending on the value of n there are four flow variation possibilities

i) n = -1 -- resolves to Free Vortex Model

ii) n= 0 ----- resolves to constant free vortex

iii) n= 1 ---- gives 'solid body rotation' model

iv) n= -2 --- produces 'strong vortex flow'



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All these flow models are considered for use in Turbine design and preliminary analysis

Next Class -----

Problem Solving on
Turbine 3-D flow theories