TURBOMACHINERY AERODYNAMICS

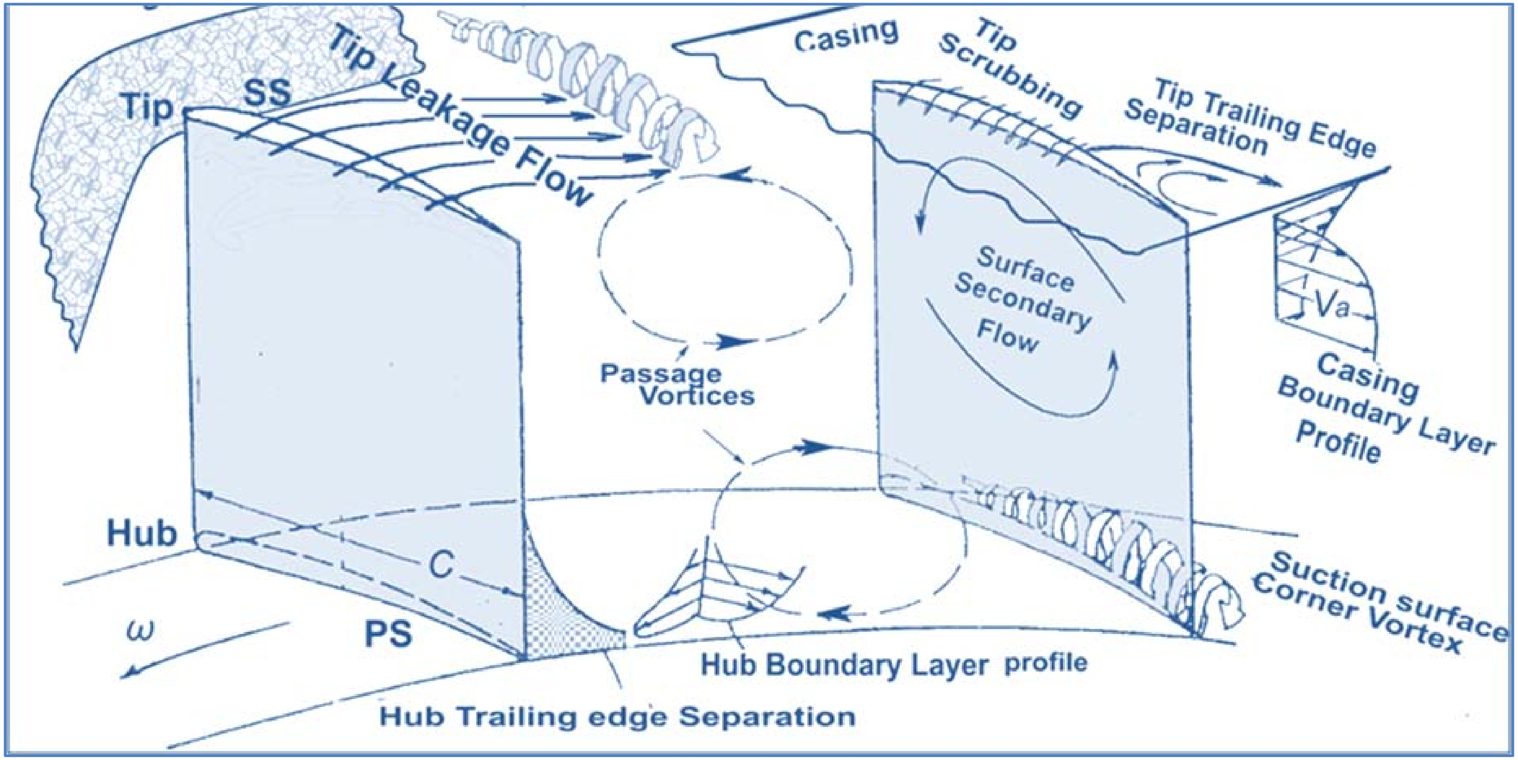
Lect - 9

Prof. Bhaskar Roy, Prof. A M Pradeep
Department of Aerospace Engineering,
IIT Bombay

Three dimensional flow analysis in Axial Flow Compressors

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

The earlier assumption on blade flow theories that the flow inside the axial flow compressor annulus is two–dimensional means that radial movement of the fluid while passing through the blade passages is ignored.



Radial flow can appear due to following reasons—

1. Centrifugal action on being imparted rotational motion is experienced by the fluid
2. Convergence of the annulus flow track introduces radiality in highly loaded compressor stages.
3. Twist and taper (chord and thickness-wise) of the blade introduces radial component to the fluid;
4. Tip clearance effects - effect of tip flow around the open tip of the blade rotor

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

5 **Passage vortex formation inside the blade passages;**

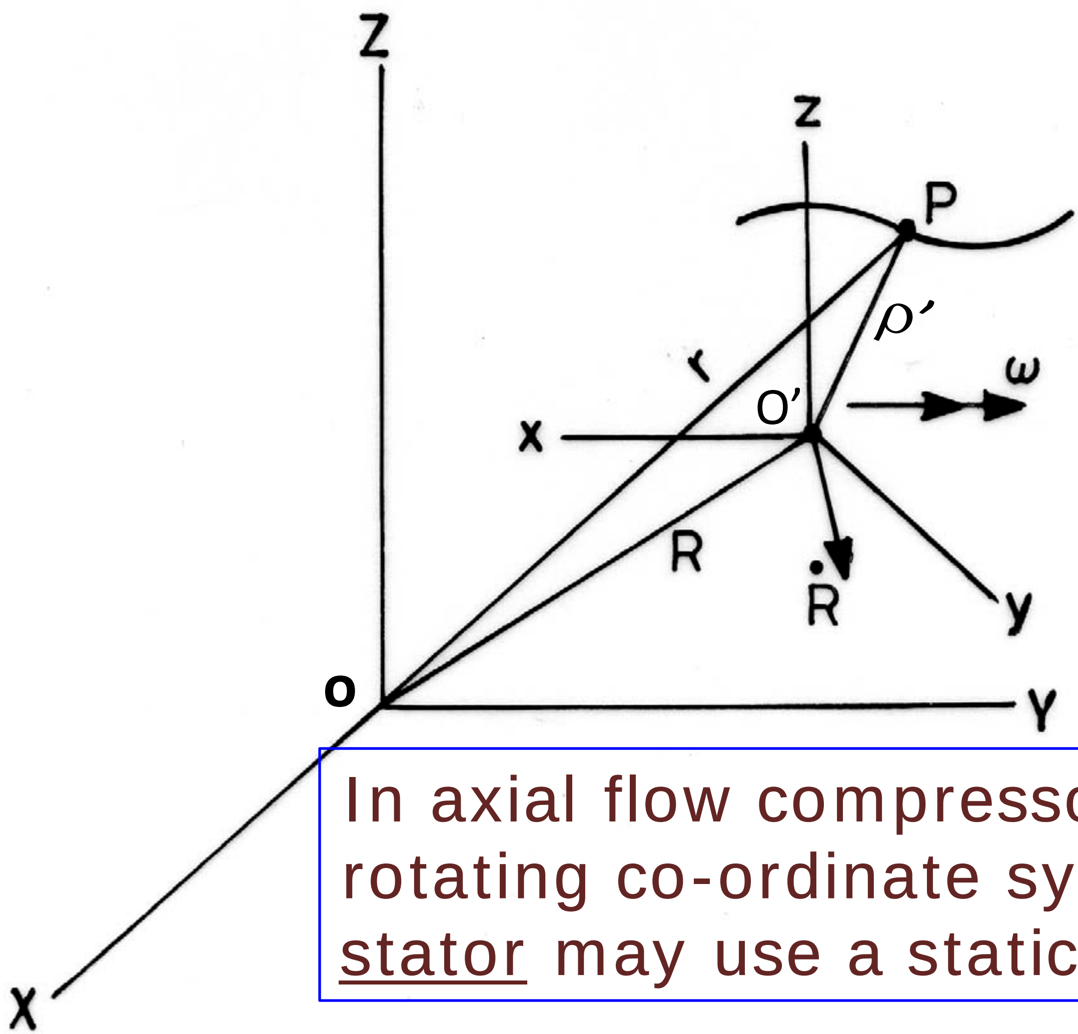
6. **Temperature/ Enthalpy / Entropy gradient in the radial direction (due to *1* to *4* above);**

7. **Blade solid body thickness blockage (including the effect of camber and stagger)**

8. **End wall (casing and hub) boundary layer blockage effects, that deflect the flow inward, in addition to reducing the main flow rate**

The radial equilibrium theory is based on the premise that the radial gradients of forces experienced by all the fluid contributes to the radial movement of the flow and hence those forces must be balanced by the static forces exerted by the pressure gradient existing in the flow, so that at any instant of time the fluid system is in radial balance of forces i.e. in radial equilibrium.

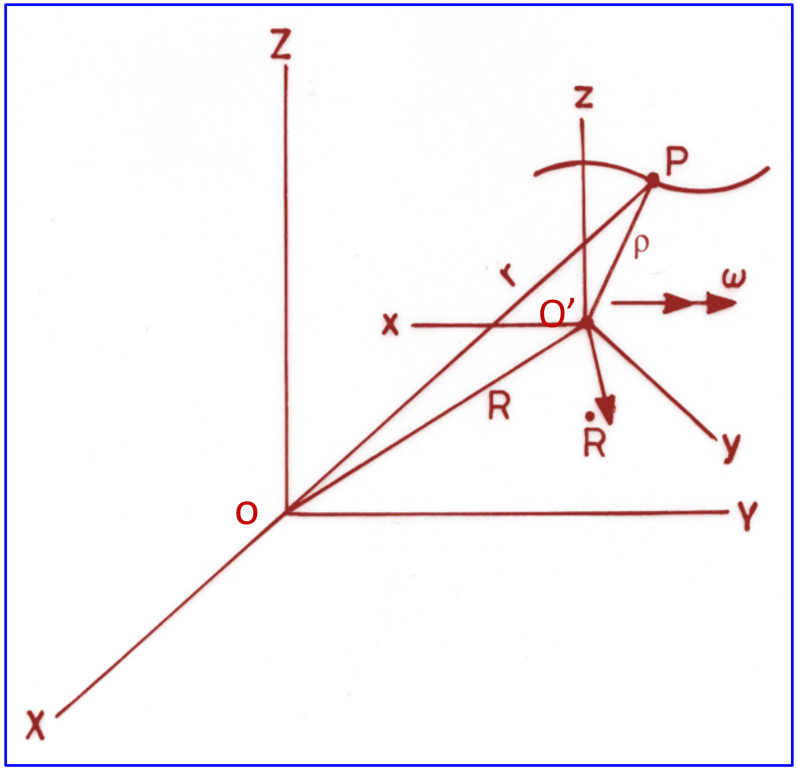
Motion of a particle w.r.t. two co–ordinate systems



Assume that a fluid particle, p is moving in an arbitrary path within the two coordinate systems

In axial flow compressors rotors need a rotating co-ordinate system, whereas the stator may use a static co-ordinate system

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay



- The two reference systems have relative motion represented by $\dot{R}$ (motion of vector R w.r.t. fixed origin, o).
- ω is rotation of the particle with respect to moving axes system xyz, and $\mathbf{v}_{xyz}$ is the translational motion of the particle with respect to moving axes xyz.

- Velocity of the particle *P* with respect to fixed axes system XYZ from the figure is :

$$V_{XYZ} = \left(\frac{d\bar{r}}{dt}\right)_{XYZ}$$

Velocity of P w.r.to small xyz, $V_{xyz} = \left(\frac{d\rho'}{dt}\right)_{xyz}$

Vectorially, motion of P is summation of the motion of the moving system w.r.t the fixed system and particle P w.r.t the moving system

$$\vec{r} = \vec{R} + \vec{\rho}\,'$$

Or,

$$\left(\frac{d\vec{r}}{dt}\right)_{XYZ} = \left(\frac{d\vec{R}}{dt}\right)_{XYZ} + \left(\frac{d\vec{\rho}\,'}{dt}\right)_{XyZ}$$

Velocity of P w.r.t the fixed system XYZ

$$V_{XYZ} = \dot{R} + V_{xyz} + \omega.\rho'$$

Definitions of the accelerations

Acceleration of P w.r.t. Fixed coordinates XYZ,

$$a_{XYZ} = \left(\frac{dV_{XYZ}}{dt} \right)_{XYZ}$$

And, acceleration of P w.r.t. Rotating coordinates xyz,

$$a_{xyz} = \left(\frac{dV_{xyz}}{dt} \right)_{xyz}$$

Thus, total acceleration of P w.r.t the fixed coordinate system XYZ,

$$a_{XYZ} = \left(\frac{dV_{XYZ}}{dt} \right)_{XYZ} =$$

$$\left(\frac{dV_{xyz}}{dt} \right)_{xyz} + \overset{**}{R} + \left[\frac{d(\omega \rho')}{dt} \right]_{XYZ}$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

The acceleration of P w.r.t the body fitted rotating coordinates xyz, is a summation of its translational and rotating motions

$$\left(\frac{dV_{xyz}}{dt}\right)_{XYZ} = \left(\frac{dV_{xyz}}{dt}\right)_{xyz} + \omega . V_{xyz}$$

Rotation in xyz system may be captured in the eqns

$$\left[\frac{d(\omega.\rho')}{dt}\right]_{XYZ} = \omega\left(\frac{d\rho'}{dt}\right)_{XYZ} + \left(\frac{d\omega}{dt}\right)_{XYZ}\rho'$$

$$\left[\frac{d\rho'}{dt}\right]_{XYZ} = \left(\frac{d\rho'}{dt}\right)_{xyz} + \omega\,\rho'$$

Acceleration of the particle p w.r.t the fixed origin O may be written as

$$a_{XYZ} = a_{xyz} + \ddot{R} + 2\omega \; v_{xyz} + \omega \left(\omega \; \dot{\rho}' \right) + \dot{\omega} \; \dot{\rho}'$$

For motion with constant angular velocity ω

$$a_{XYZ} = a_{xyz} + 2\omega \; v_{xyz} + \omega \left(\omega \; \rho' \right)$$

For a compressor blade passage, the flow velocities are

$$V_{xyz} = V \quad (relative\ velocity)$$

$$V_{XYZ} = C \quad (absolute\ velocity)$$

$$V_{XYZ} = V_{xyz} + \omega\rho' = V + \omega r = V + u$$

Differentiating,

$$a_{XYZ} = \left[\frac{d(V_{XYZ})}{dt}\right]_{XYZ} - \left[\frac{d(\omega\rho')}{dt}\right]_{XYZ}$$

Translational motion — Rotation

Final accn.

$$a_{XYZ} = a_{xyz} + 2\omega\,V_{xyz} + \omega\,(\omega\,\rho')$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

14

Considering the equilibrium of forces along arbitrary flow direction, *s* – direction, we get, between any two axial stations, separated by a small distances Δs where area, A_i is constant.

Pr. Force, $\Delta p . A_i = A_i . \rho . \Delta s . a_{XYZ}$ (Mass x acceleration)

Hence, $$\frac{1}{\rho}\frac{\Delta p}{\Delta s} = a_{XYZ}$$

Thus, the acceleration equation from the last slide may be written as :

$$\frac{1}{\rho}.\nabla p = \omega^2 \frac{Dv}{Dt} r + 2\omega V$$

As the flow in compressor blade is diffusing

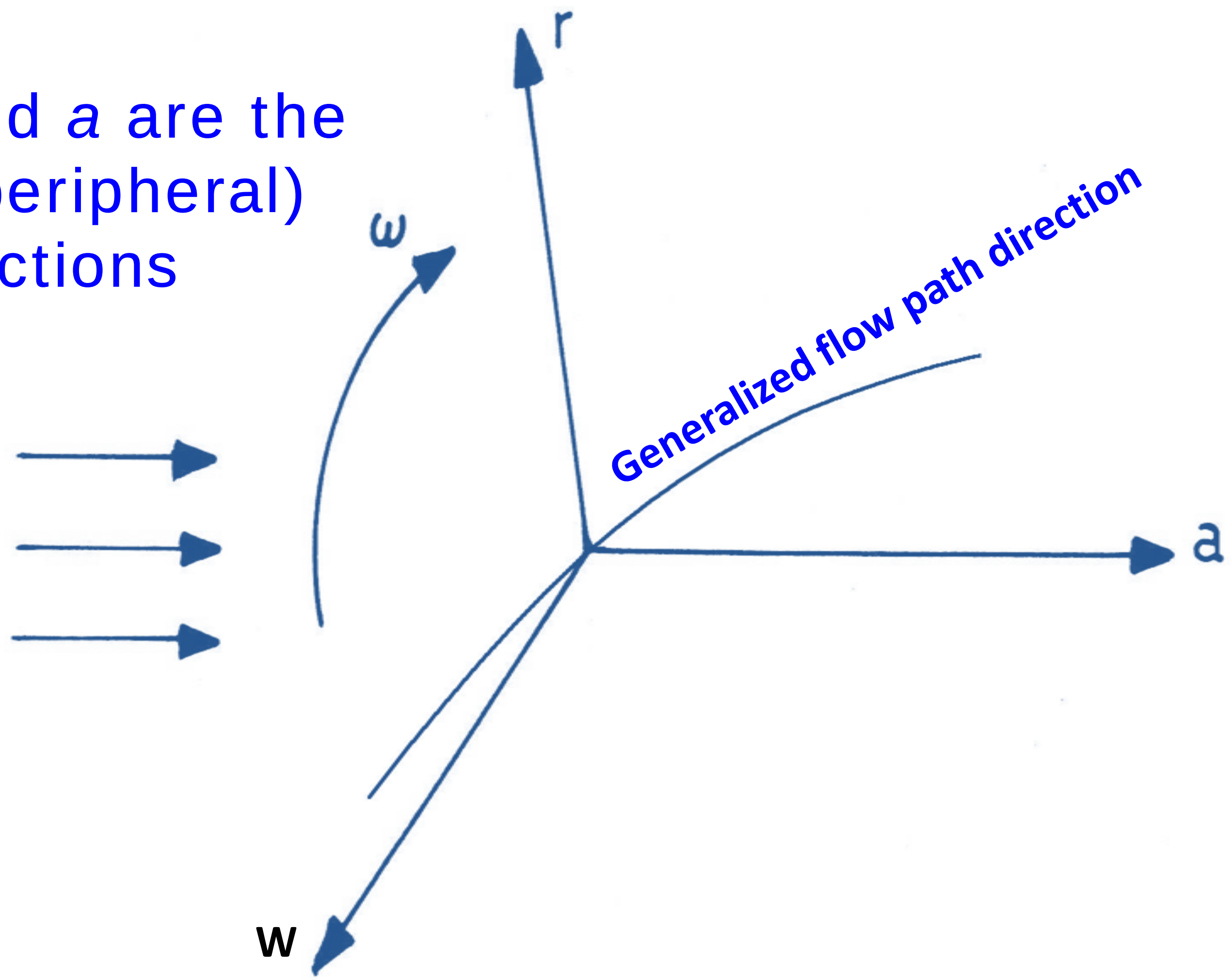
$$\frac{DV}{Dt} \quad \text{is negative}$$

$$-\frac{1}{\rho}.\nabla p = \omega \frac{Dv}{Dt} r + 2^2 \omega \ V \times \quad \times$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

New of axis notations

where *r*, *w* and *a* are the radial, whirl(peripheral) and axial directions respectively.



Assumptions made are:-

- The fluid is frictionless
- The rotor is rigid and rotates with constant angular velocity ω
- The flow is steady relative to the rotor
- The radial variation of density is neglected

This still leaves enough scope for formation of

i) vorticity,

ii) entropy gradients, *and*

iii) stagnation enthalpy gradients in the flow field.

Then from the definition of unit vectors

$$\frac{D\hat{i}_r}{Dt} = 3\hat{i}_r = \frac{V_r}{r}\hat{i} \quad \text{and} \quad \frac{D\hat{i}_w}{Dt} = -3\hat{i}_w = -\frac{V_w}{r}\hat{i}_r$$

using, $$V = V_r\hat{i}_r + V_w\hat{i}_w + V_a\hat{i}_a$$

By definition $$\frac{D(\quad)}{Dt} = V\frac{D(\quad)}{Ds} + \left(\frac{ds}{dt}\right)$$

$\frac{ds}{dt} \rightarrow 0$

s is length in any direction

as $$\frac{ds}{dt} = 0$$ For Steady State flow

Now, the equation for flow inside the compressor blade passage may be recast using r, θ and z coordinate system and modified to :

$$\frac{DV}{Dt} = \hat{i}_r\left(\frac{DV_r}{Dt} - V_w\frac{d}{dt}\right) + \hat{i}_w\left(\frac{DV_w}{Dt} + V_r\frac{}{dt}\right) + \hat{i}_a\frac{DV_a}{Dt}$$

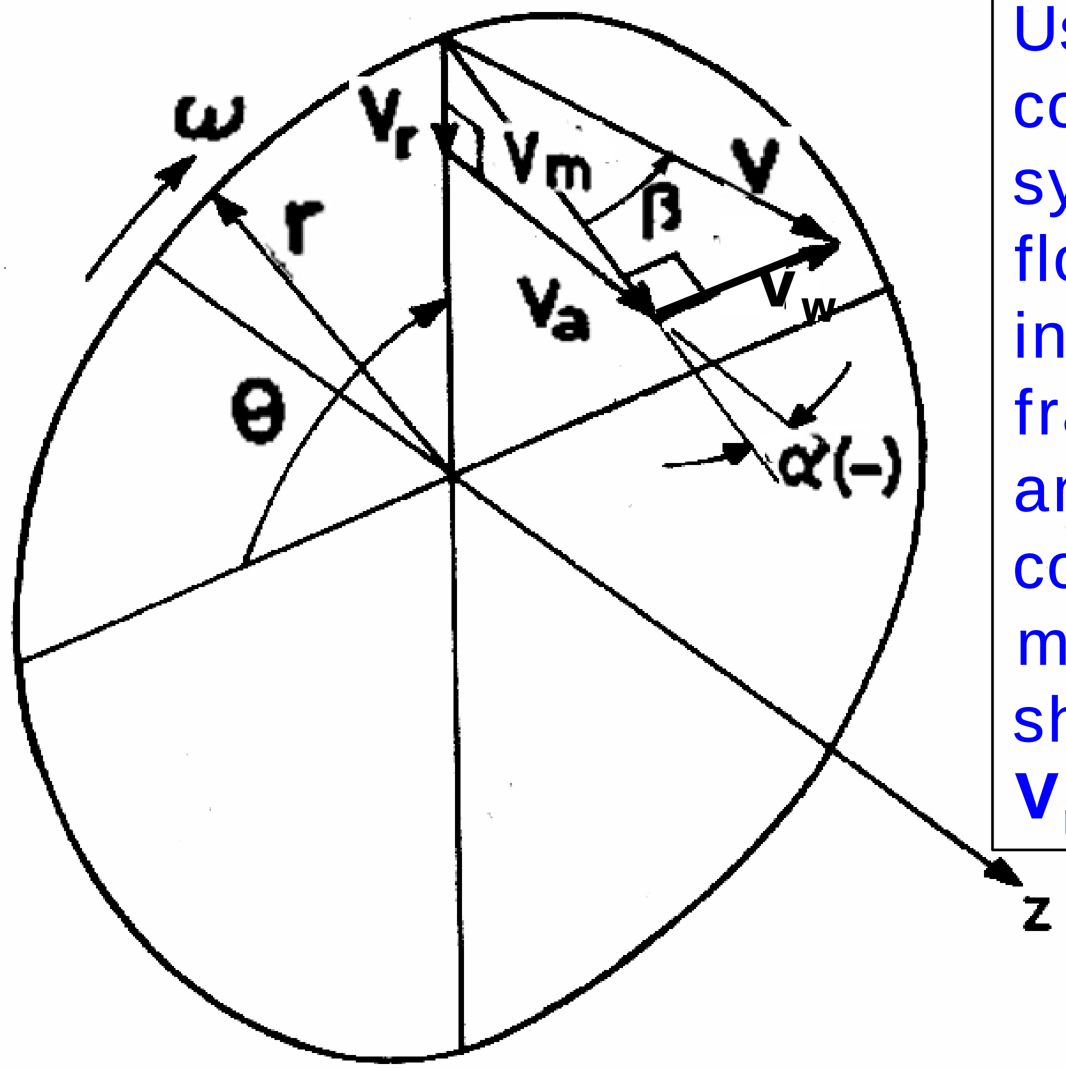
Using the coordinate systems the flow velocity in relative frame is **V** and its components may be shown as $\mathbf{V_a}$, $\mathbf{V_r}$, $\mathbf{V_w}$

Now, the equation may be resolved in its three components, using r, θ and z coordinate system,

$$-\frac{1}{\partial \varphi}\frac{}{} = V\left(\frac{D V_r}{}\right) - \frac{\left(V_w + r3\right)^2}{r} \qquad \text{---------- (a)}$$

$$-\frac{1}{\partial \varphi}\cdot\frac{}{} = V\left(\frac{D V_a}{}\right) + \frac{V_w V_r}{r} + 2.\quad r \qquad \text{-------- (b)}$$

$$-\frac{1}{\partial \varphi}\frac{}{} = V\left(\frac{D V_a}{}\right) \qquad \text{-------------- (c)}$$



Using the coordinate systems the flow velocity in relative frame is **V** and its components may be shown as $\mathbf{V_a}$, $\mathbf{V_r}$, $\mathbf{V_w}$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

The velocity triangle for the flow gives us

$$C_w = V_w +$$

Then equation (a) and (b) from slide 21 can be rewritten as

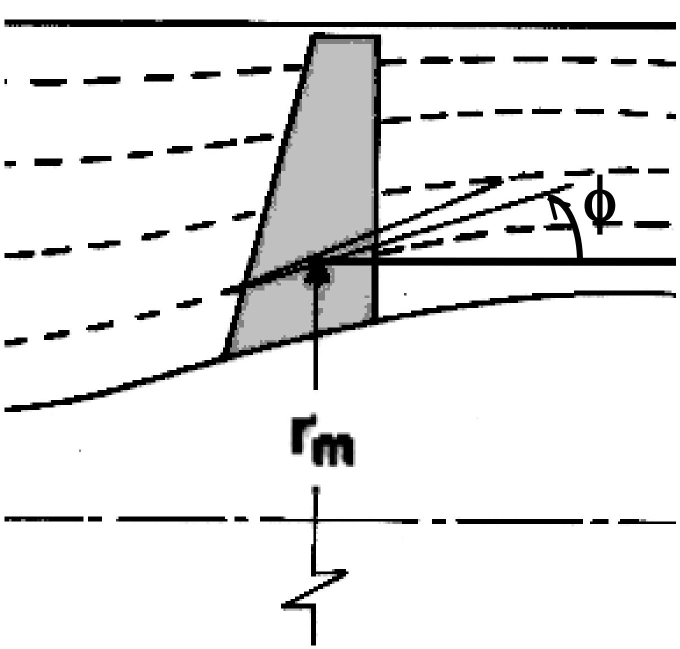
$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = V\left(\frac{DV_r}{Ds}\right) - \frac{\left(C_w\right)^2}{r} \qquad \text{---------- (d)}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} = \frac{V}{r}\left(\frac{D\left(r.C_w\right)}{Ds}\right) \qquad \text{----------- (e)}$$

23

Now, we can write the kinematic relation as,

$$V\frac{D(\quad)}{Ds} = V_a\frac{D(\quad)}{Da}$$



Where V_a and a are axial the components of V and s respectively. Define a meridional direction by

$$D_m\hat{i}_m = D_r\hat{i}_r + D_a\hat{i}_a$$

Now, equation (d) from the last slide can be re-written as :

$$-\frac{1}{\partial p\rho}\,\overline{} = V_m\left(\frac{DV_r}{}\right) - \frac{(c_w)^2}{r}$$

24

Meridional direction may be defined as

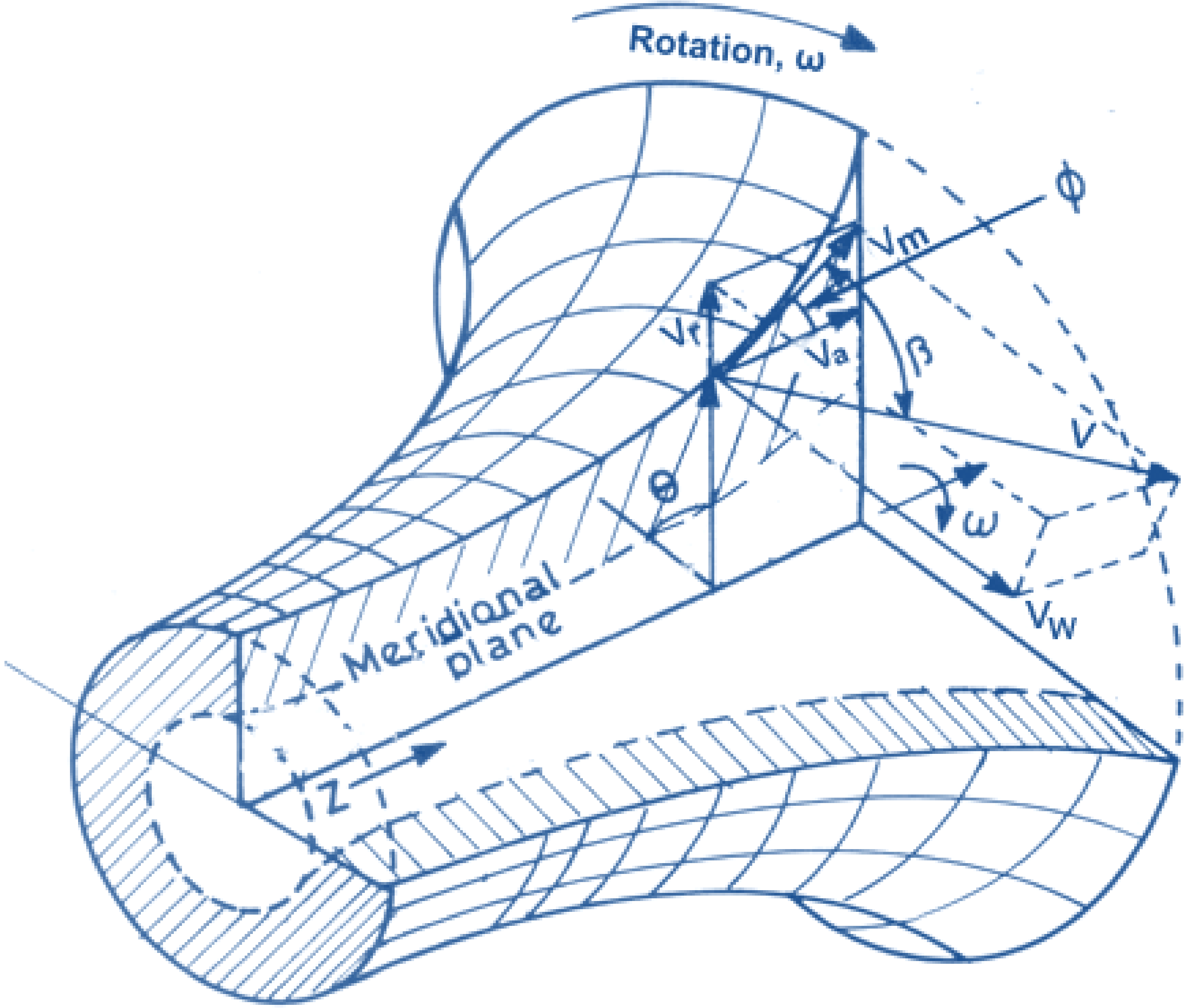
$$\tan. = \theta \quad \frac{V_r}{V_z} \qquad \text{and} \qquad V = V_m \sin\varphi$$

Hence the force balance equation only in the radial direction is re-written as :

$$\frac{1}{\partial p}\,\frac{}{\rho\, r} = \frac{C_w^2}{\theta} - V_m^2 \frac{}{D\sin.} - V_m \sin. \frac{DV_m}{}$$

Now, by our earlier definition

$$V \frac{D(\quad)}{Ds} = V_m \frac{D(\quad)}{Dm}$$



Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Now, $\frac{D\,\sin.\phi}{Dm} = \cos.\phi\,\frac{D\,\phi}{D\,m}$ **and** $\frac{D\phi}{Dm} = -\frac{1}{r_m}$

Where **r_m is the radius of curvature of the meridional flow**

The negative sign is arbitrary. But, for axial flow compressor the flow track inside generally moves towards lesser ϕ or higher r_m, i.e. the flow later on flattens out. Hence,

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_w^2}{r} + \cos.\phi\,\frac{V_m^2}{r_m} + V_r\,\frac{DV_m}{Dm}$$

This is the *full radial – equilibrium Equation* for circumferentially averaged (blade to blade) flow properties inside of a turbo machine blade row

For old fashioned compressor designs V_m (instead of constant axial velocity V_a / C_a) is considered constant and the last term is eliminated. In the very early design of compressor the flow path was considered linear and hence even the 2nd term vanishes, giving us back the simple radial equilibrium equation

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_w^2}{r}$$

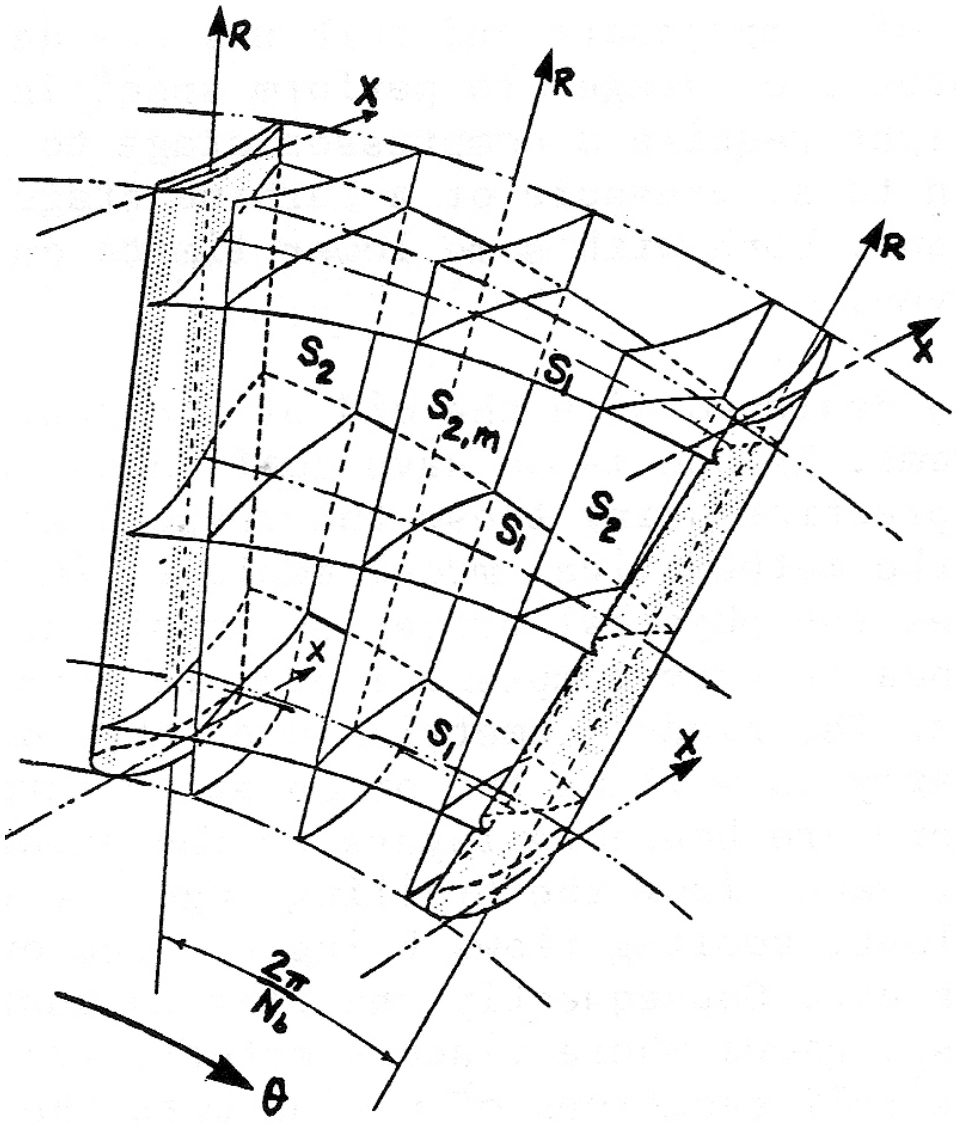
For modern compressor, this simple radial equilibrium equation relationship is inadequate and it becomes necessary to utilize the full radial equlibrium equation.

- **Wherever the flow is not experiencing the centrifugal force, the radial equilibrium can not be applied.**
- **Experiments have shown that in between the blade rows, in the axial gaps between the rotor and the stator, there could be radial shift of the meridional path. Hence for accurate design flow analysis the full radial equilibrium equation is be used.**
- **For using the full REE, for computational purposes, further steps need to be taken.**

i) The R.E.E is to be transformed into a form that contains partial derivatives of all parameters with respect to r and θ

ii) Next, the circumferential average of those parameters is taken by integrating over θ from pressure side of one blade to the suction side of the other blade.

iii) The flow is analysed at various axial stations with a) Energy equation, b) Continuity condition and c) R.E.E.


R
X
R
R
X
S2
S1
S2,m
S2
S1
X
S1
X
2π
Nb
θ

- It is necessary that flow properties obtained in this manner at various axial stations be consistent with one another as the flow properties are evaluated from hub to tip at each station.
- That means radial acceleration of the fluid particle is to be accounted for in the R.E.E.
- This can be achieved by assuming shapes for the meridional streamlines consistent with the continuity condition expressing the radial acceleration in terms of the streamline slope and curvature.
- This implies an iterative method of solution.
- This method, in which surfaces are used to build up flow inside a turbomachinery blade, has been widely used. The equations on the blade-to-blade surface and those on the meridional plane need to be solved separately.

- The 3-D flow computations has provided immense assistance to engine designers.
- It has cut down on design time and has reduced dependence on costly experimental analysis.
- The 3-D methods have helped understand various flow phenomena e.g. secondary flow development, chocking in the stages, effects of end-wall flows etc.
- However, the designer uses these solutions in conjunction with many empirical relations and experimental data to make the design.
- There is still scope for improvement in these methods and for reducing dependence on empirical relations

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Next Class

Problem Solving and Tutorial problems using
simple 3-D flow theories
on
Axial Flow Compressor

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay