



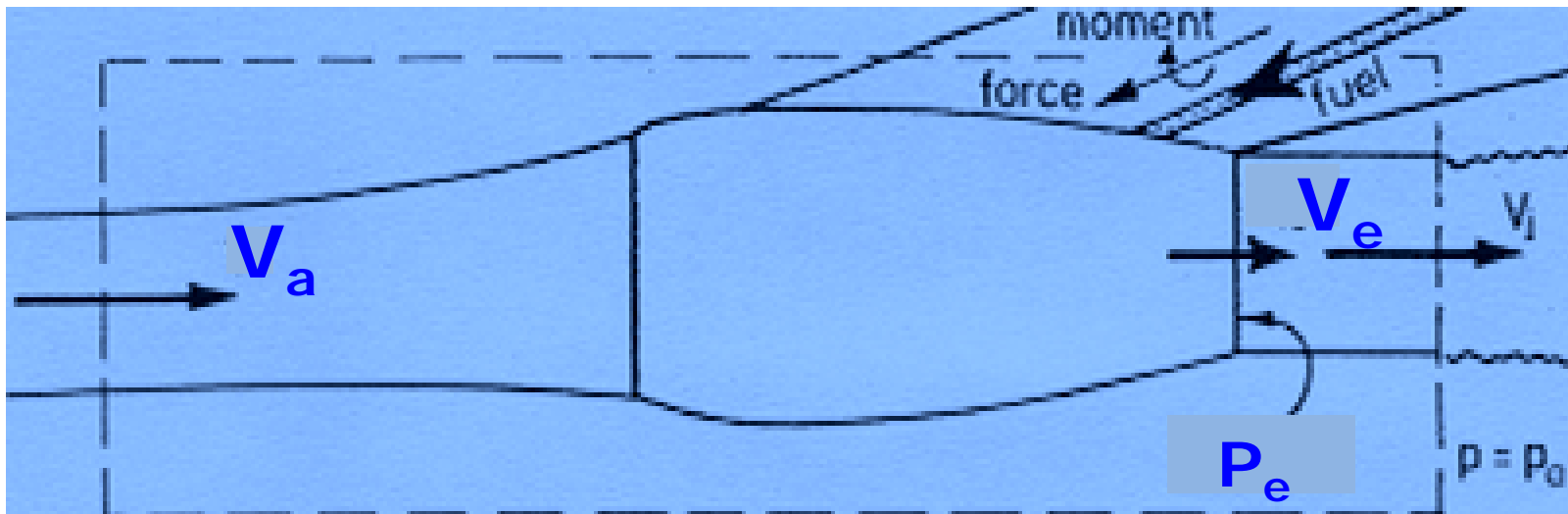
Jet Aircraft Propulsion

Prof. Bhaskar Roy, Prof. A M Pradeep

Department of Aerospace Engineering,
IIT Bombay

Lecture 33

Matching of Engine Components



- Any variable in the engine is expressed as a function of \dot{m}_f , P_a , T_a and V .
- Because the engine is self contained; fixing one set of engine non-dimensional parameters fixes all others.

- Instantaneous cycle temperature ratio, T_{03}/T_{01} fixes the cycle pressure ratio π_c , the instantaneous fuel flow \dot{m}_f and the instantaneous rotational speed, n .
- Similarly in a multi-shaft engine the ratio between the shaft speeds, n_2/n_1 is also fixed by T_{03}/T_{01} .
- For most operating conditions of the engine the final propelling nozzle for the core flow and the bypass flow will be choked, by design, for maximizing, thrust.
- Thus the engine responds to the inlet stagnation conditions but is unaware of the forward speed.

Non-dimensional Variables of the Engine

1) Consider mass flow of air through the engine.

This can be written,

as a function of the rotational speed N of the shafts

$$\dot{m}_a = f(N, P_{01}, T_{01})$$

or, in terms of turbine inlet temperature

$$\dot{m}_a = f(T_{03}, P_{01}, T_{01})$$

or, in terms of fuel flow rate

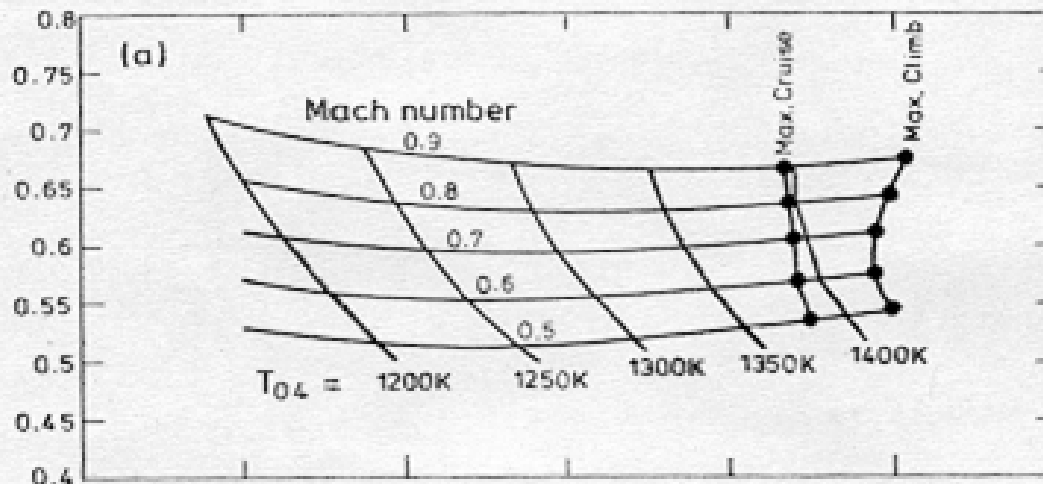
$$\dot{m}_a = f(\dot{m}_f, P_{01}, T_{01})$$

The non-dimensional mass flow rate is derived from dimensional analysis (*e.g. Buckingham Π Theorem*), as

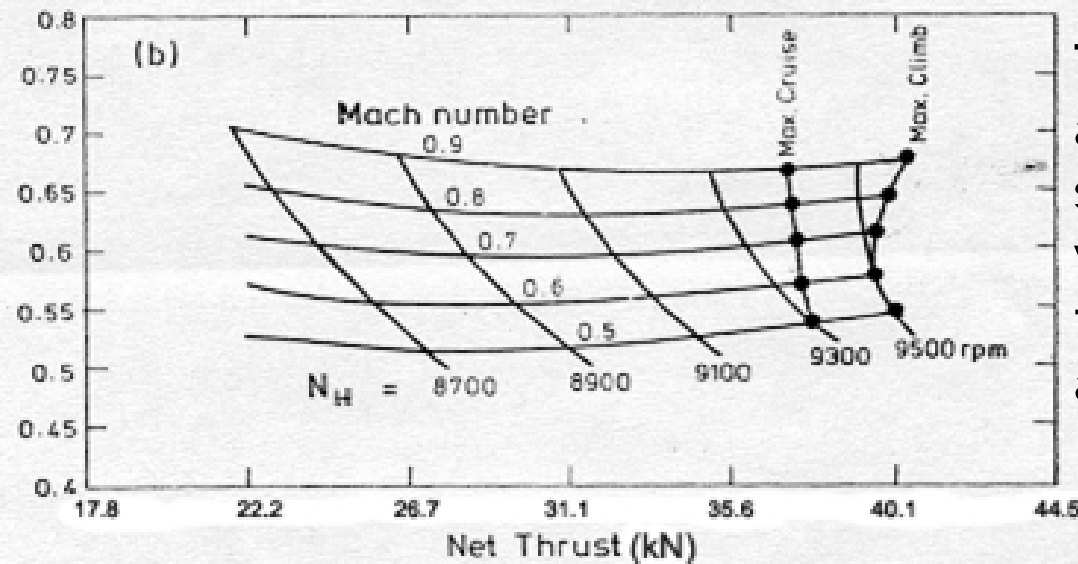
$$\overline{\dot{m}} = \frac{\dot{m}_a \cdot \sqrt{(c_p \cdot T_{01})}}{D^2 \cdot P_{01}}$$

Where, D is a characteristic diameter of the engine, typically the diameter of the inlet of the fan and D^2 denotes a representative area

sfc kg/hour/kg



Functional dependence of engine parameters is seen in engine characteristics



The lines of constant T_{04} and constant rpm are assumed parallel, so that once one variable is chosen the other variables are also fixed.

Practical Normalizing of Parameters

For mass flow of air c_p is constant and D^2 is constant for a given engine.

$$\overline{\dot{m}} = \frac{\dot{m}_a \cdot \sqrt{T_{01}}}{P_{01}}$$

it has units.

Normalized fuel flow the abbreviated form is derived,

$$\overline{\dot{m}}_f = \frac{\dot{m}_f}{\sqrt{T_{03}} \cdot P_{03}}$$

Non-dimensional speed is abbreviated : $N/(T_{02})^{1/2}$

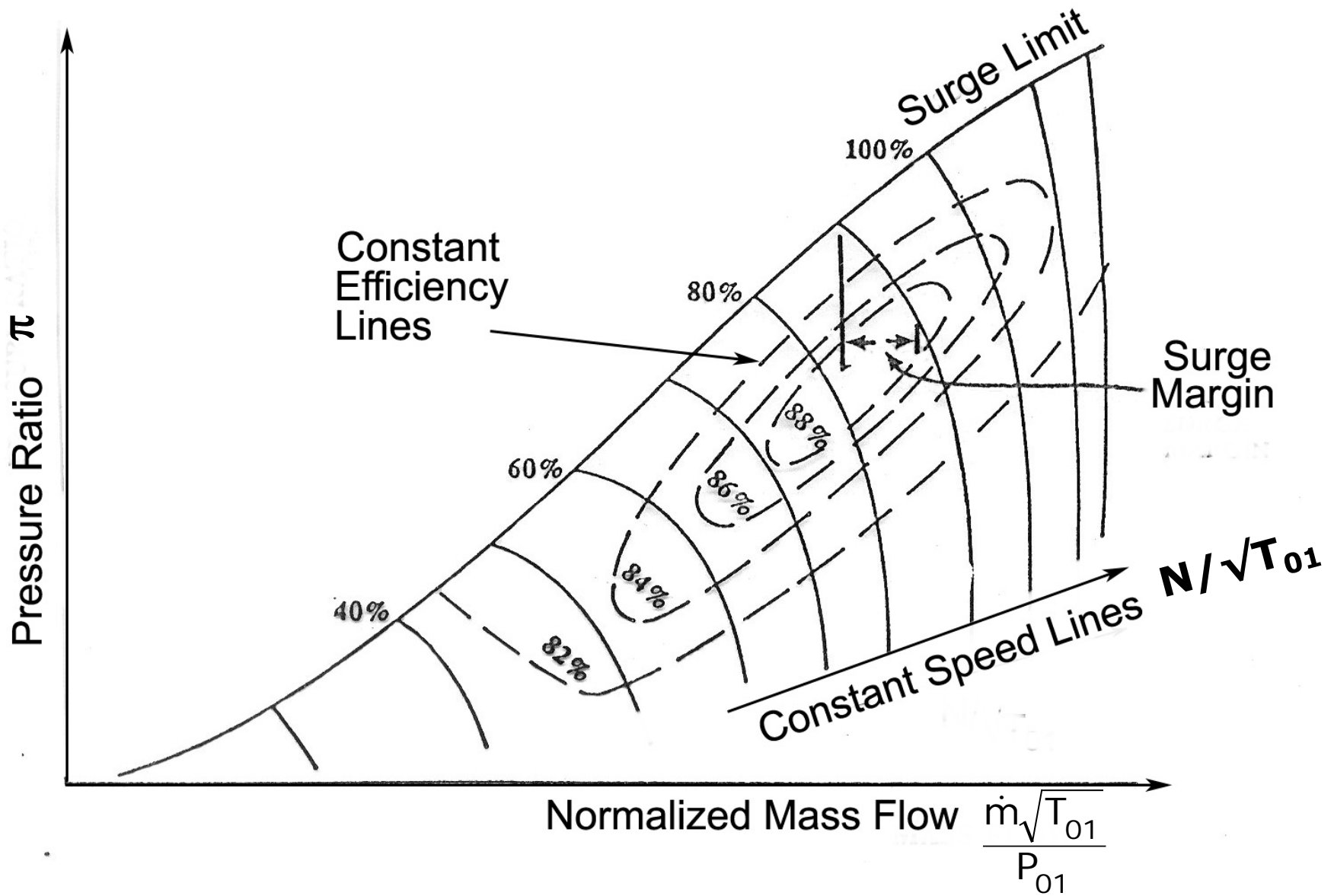
The corrected mass flow (kg/s) is $\overline{\dot{m}} = \frac{\dot{m}_a \cdot \Theta}{\delta}$

Where. $\Theta = T_{02}/T_{02ref}$, $\delta = P_{02}/P_{02ref}$,

and T_{02ref}, P_{02ref} are STP

Matching Procedure :

- 1) Select operating point (Altitude, Flight Condition)
- P_a , T_a , and M
- 2) From the ambient condition obtain - P_{01} , T_{01}
- 3) Select max Turbine entry temp. T_{03}
- 4) Select Rotational speed, N – then obtain
- 5) Obtain - $N/\sqrt{T_{01}}$, and $N/\sqrt{T_{03}}$
- 6) Select a compressor pressure ratio , $\pi_{0c} = P_{02}/P_{01}$
- 7) Obtain mass flow parameter,
$$\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}$$
- 8) The parameters in (5), (6) and (7) completely define the compressor operation point



9) Actual mass flow through the compressor is :

$$\dot{m}_c = \frac{\dot{m} \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{T_{01}}$$

10) The turbine mass flow is :

$$\dot{m}_T = \dot{m}_c \cdot (1 + f - b)$$

where, f - is fuel/air ratio, b = air bleed

and Turbine entry Pressure, $P_{03} = P_{02} (1 - \Delta P_{cc})$

11) Based on the actual mass flows through the compressor and the turbine, the work done for these mass flows need to be equated (turbojet engine). The same may be done spool-wise.

12) Work equivalence :

$$\dot{m}_C \cdot \overline{W}_C = \dot{m}_T \cdot \overline{W}_T \cdot \eta_{\text{mech}}$$

for a turbojet engine, or for a spool, i.e.

$$\dot{m}_{\text{LPC}} \cdot \overline{W}_{\text{LPC}} = \dot{m}_{\text{LPT}} \cdot \overline{W}_{\text{LPT}} \cdot \eta_{\text{mech-LP}} \quad \text{for LP spool}$$

$$\dot{m}_{\text{Fan}} \cdot \overline{W}_{\text{Fan}} = \dot{m}_{\text{LPT2}} \cdot \overline{W}_{\text{LPT2}} \cdot \eta_{\text{mech-FT}} \quad \text{for fan-turbine}$$

Where η_{mech} is the
Mechanical efficiency of
the shaft

or

$$\dot{m}_{\text{HPC}} \cdot \overline{W}_{\text{HPC}} = \dot{m}_{\text{HPT}} \cdot \overline{W}_{\text{HPT}} \cdot \eta_{\text{mech-HP}} \quad \text{for HP spool}$$

13) Compressor /Fan specific work (per unit mass flow)

$$\overline{W}_c = \frac{C_{p\text{-air}} \Delta T_{012}}{\eta_c} = \frac{C_{p\text{-air}} T_{01}}{\eta_c} \left[\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma_{\text{air}} - 1}{\gamma_{\text{air}}}} - 1 \right]$$

14) Turbine Specific work (per unit mass) :

$$\overline{W}_T = \eta_T \cdot C_{p\text{-gas}} \Delta T_{034} = \eta_T C_{p\text{-gas}} T_{03} \left[1 - \frac{1}{\left(\frac{P_{03}}{P_{04}} \right)^{\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}}} \right]$$

15) Turbine mass flow parameter can be determined from :

$$\frac{\dot{m}_{\text{gas}} \sqrt{T_{03}}}{P_{03}} = \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{P_{02}} \cdot \frac{P_{02}}{P_{03}} \cdot \frac{\dot{m}_{\text{gas}}}{\dot{m}_{\text{air}}} \sqrt{\frac{T_{03}}{T_{01}}}$$

16) The net turbine – compressor excess power (if any) may now be decided by:

$$P_{\text{engine}} = \dot{m}_{\text{gas}} \cdot \overline{W}_T - \frac{\dot{m}_{\text{air}} \cdot \overline{W}_c}{\eta_{\text{mech}}}$$

Where η_{mech} is the Mechanical efficiency of the shaft

For pure turbojet $P_{\text{engine}} = 0$,

For multi-spool turbofan $P_{\text{spool}} = 0$

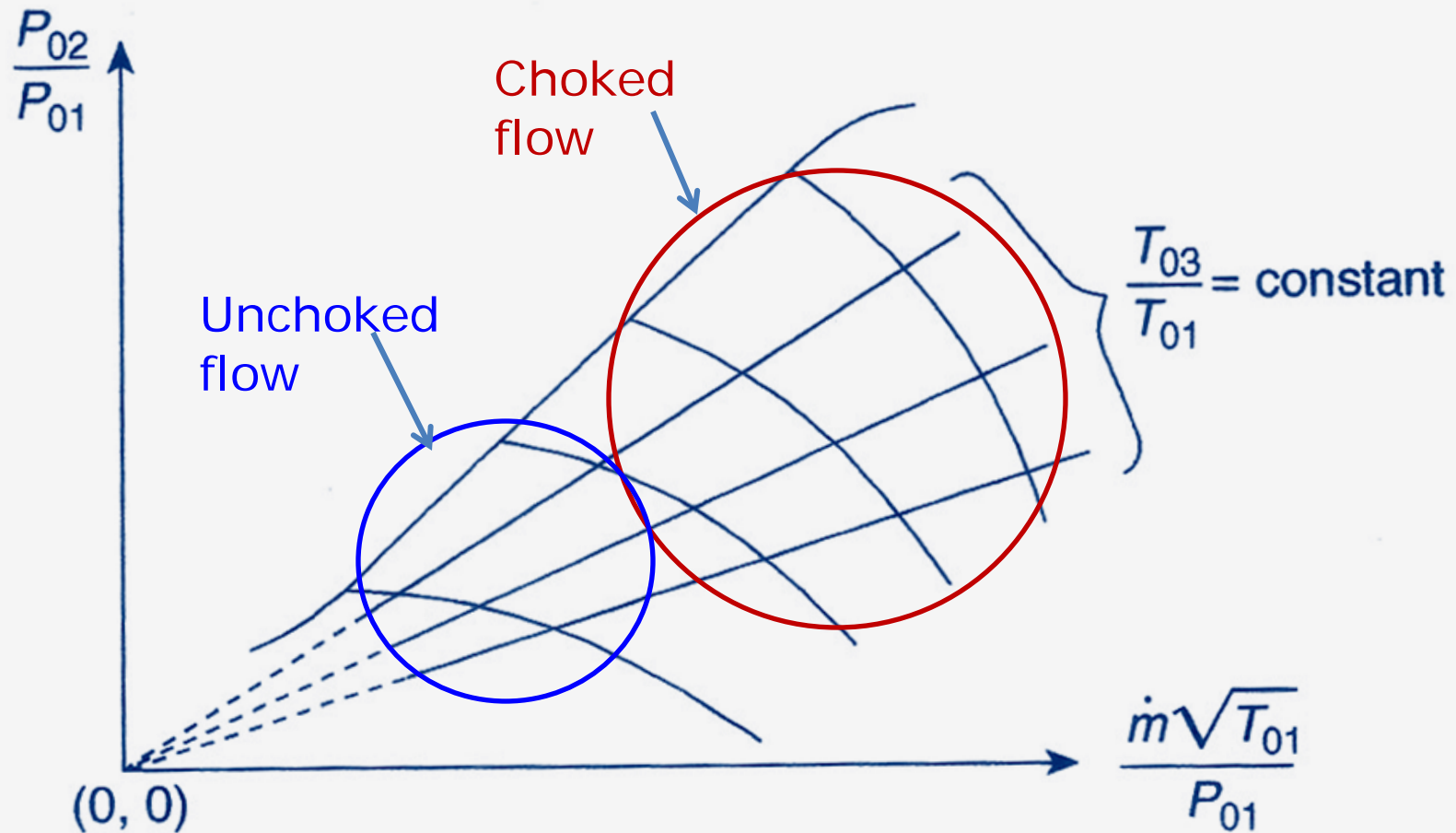
- The power or work matching between compressor and turbine has to be exact.
- If they are unequal, either a new speed of the engine or new values of the **compressor pressure ratio & mass flow** are selected to try and arrive at perfect matching.
- For turboprop or turboshaft the excess shaft power $P_{\text{engine}} = P_{\text{propeller/rotor}}$ and requires exact matching.
- If they are unequal – either the engine speed setting (N) or mass flow setting (\dot{m}), or compression ratio π_c , or TET, T_{03} is to be selected --- or -- a new propeller/ rotor pitch setting needs to be selected.
- Logic of this selection has to be built in to the control system logic of the engine (FADEC)

In case of a choked nozzle, the actual mass flow is invariant with change of other parameters:

$$\frac{\dot{m}_{\text{gas}} \sqrt{T_{03}}}{P_{03}} = \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{P_{02}} \cdot \frac{P_{02}}{P_{03}} \cdot \frac{\dot{m}_{\text{gas}}}{\dot{m}_{\text{air}}} \sqrt{\frac{T_{03}}{T_{01}}} = \text{const}$$

Assume that pressure ratio across the combustion chamber, $P_{02}/P_{03} = \text{constant}$ and $\dot{m}_{\text{air}} = \dot{m}_{\text{gas}}$

$$\frac{P_{02}}{P_{01}} = K_1 \cdot \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \sqrt{\frac{T_{03}}{T_{01}}}, \quad \text{where } K_1 \text{ is a constant}$$



Constant turbine inlet temperature on the compressor map.

assume that cycle temperature ratio is constant, then

$$\frac{P_{02}}{P_{01}} = K_2 \cdot \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}}, \quad \text{where } K_2 \text{ is another constant}$$

For a straight and level cruise flight

$$\frac{P_{02}}{P_{01}} = K_3 \cdot \sqrt{\frac{T_{03}}{T_{01}}}, \quad \text{where } K_3 \text{ is another constant}$$

If T_{01} / T_{03} is held constant for a cruise flight

$$\frac{P_{02}}{P_{01}} = K_4, \quad \text{where } K_4 \text{ is another constant}$$

Off-Design Matching of Turbojet Engine

The Intake delivery Pressure and Temperature can be found from

$$\frac{P_{01}}{P_a} = \left(1 + \eta_I \cdot \frac{\gamma_{air} - 1}{2} \cdot M_a^2 \right)^{\frac{\gamma_{air}}{\gamma_{air} - 1}}$$

$$\frac{T_{01}}{T_a} = \left(1 + \frac{\gamma_{air} - 1}{2} \cdot M_a^2 \right)$$

- Higher the flight Mach number, M_a higher would be P_{01} and T_{01} at any constant altitude
- For constant flight Mach number, M_a - the P_{01} and T_{01} decreases with increase of altitude & vice versa

- The Ram pressure development in the intake increases/ decreases the compressor inlet and then compressor outlet pressure.
- It then increases / decreases the turbine inlet / outlet pressure
- Thus the pressure ratio across the nozzle increases / decreases
- At high nozzle pressure ratio, the flow is choked – it is then independent of nozzle pressure ratio – and hence from flight speed
- This fixes the turbine operating point w.r.t nozzle choked condition.

An aero engine is choked most of the time of flight except during – Taxiing, Approaching and Landing

The nozzle pressure ratio may be computed from

$$\frac{P_{04}}{P_a} = \frac{P_{04}}{P_{03}} \cdot \frac{P_{03}}{P_{02}} \cdot \frac{P_{02}}{P_{01}} \cdot \frac{P_{01}}{P_a}$$

Where, P_{01}/P_a is obtained from Intake analysis

Thrust of the engine may be written as:

$$F = m_5 (V_5 - V_{flight}) + A_5 (P_5 - P_a),$$

where flight speed is given by, $V_{flight} = M_a \sqrt{\gamma \cdot R \cdot T_a}$

The gas exhaust speed V_5 depends on the flow condition at nozzle exit, and with choking reaches the maximum for that condition. For example, some time during the climb operation if the nozzle is choked, it will remain so during the climb, with continuous fall in ambient pressure P_a .

For choked nozzle:

$$F = m_5 (V_5 - V_{flight}) + A_5 (P_5 - P_a),$$

where exhaust velocity is given by,

$$V_5 = \sqrt{\gamma_{gas} \cdot R \cdot T_5} = \sqrt{\frac{2\gamma_{gas} \cdot R \cdot T_{04}}{\gamma_{gas} + 1}}$$

For un-choked nozzle :

$$V_5 = \sqrt{\gamma_{gas} \cdot C_{p-gas} \cdot (T_{04} - T_5)} = \sqrt{2C_{p-gas} \cdot T_{04} \left[1 - \left(\frac{P_a}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

For choked nozzle:

$$P_5 = P_c = P_{04} \left(1 - \frac{1}{\eta_{nozzle}} \cdot \frac{\gamma_{gas} - 1}{\gamma_{gas} + 1} \right)^{\frac{\gamma_{gas}}{\gamma_{gas} - 1}}$$

For un-choked nozzle :

$$P_5 = P_a$$

- The engine performance seems to be decided by engine normalized speed $N/\sqrt{T_{01}}$, but the maximum performance is capped by the engine speed, N_{max} .
- This max speed is decided by stress limits of rotating components.
- At $N_{max}/\sqrt{T_{01}}$ as the ambient temperature increases thrust will decrease, but engine speed cannot be increased much more.
- The engines are often designed for 15°C , 288K . They are bound to give lower performance in tropical atmospheres & higher performance in colder climates.

Next class :

Engine Component Matching and Sizing