Jet Aircraft Propulsion

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Lect-9

In the next lecture…

- Tutorial on ideal cycles and component performance.

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Problem # 1

- A Brayton cycle operates with a regenerator of 75% effectiveness. The air at the inlet to the compressor is at 0.1 MPa and 30°C, the pressure ratio is 6.0 and the maximum cycle temperature is 900°C. If the compressor and the turbine have efficiencies of 80% each, find the percentage increase in the cycle efficiency due to regeneration.

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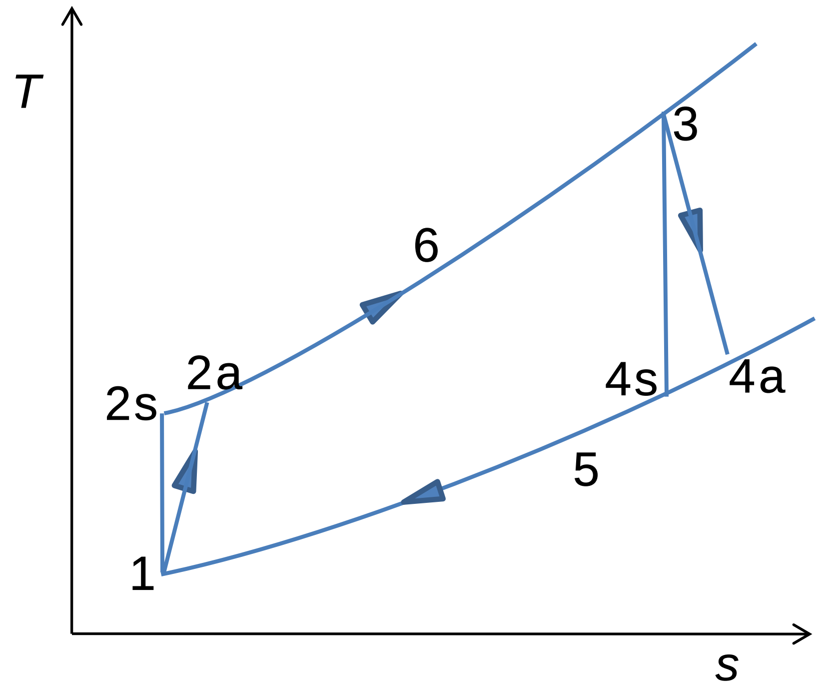
Problem # 1



$P_{01} = 0.1$ MPa
$T_{01} = 303$ K
$T_{03} = 1173$ K
$\pi_c = 6$
η_c, $\eta_t = 0.8$

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Solution: Problem # 1

- Let us first consider the Brayton cycle without the regenerator:

$$\frac{T_{02s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma} = \frac{T_{03}}{T_{04s}} = 6^{0.4/1.4} = 1.668$$

$$T_{02s} = 303 \times 1.669 = 505K$$

$$T_{04s} = \frac{1173}{1.668} = 705K$$

From the definition of isentropic efficiency of a compressor,

$$T_{02} - T_{01} = \frac{T_{02s} - T_{01}}{\eta_C} = \frac{505 - 303}{0.8} = 252K$$

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Solution: Problem # 1

For a turbine we have,

$$T_{03}-T_{04}=\eta_t(T_{03}-T_{04s})=0.8(1173-705)=375K$$

Therefore,

$$w_t=h_{03}-h_{04}=c_p(T_{03}-T_{04})=1.005\times 375=376.88\,\text{kJ/kg}$$

$$w_c=h_{02}-h_{01}=c_p(T_{02}-T_{01})=1.005\times 252=253.26\,\text{kJ/kg}$$

Now, $T_{02}=252+303=555K$

Hence, $Q_1=h_{03}-h_{02}=1.005\times(1173-555)$

$$=621.09\,\text{kJ/kg}$$

$$\therefore \eta=\frac{w_t-w_c}{Q_1}=\frac{376.88-253.26}{621.09}=19.9\%$$

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Solution: Problem # 1

Let us now consider the regenerator :

$$T_{04} = T_{03} - 375 = 1173 - 375 = 798K$$

$$\text{Regenerator effectiveness} = \frac{T_{06} - T_{02}}{T_{04} - T_{02}} = 0.75$$

$$\therefore T_{06} - 555 = 0.75(798 - 555)$$

$$or, T_{06} = 737.3K$$

$$\text{Now}, Q_1 = h_{03} - h_{06} = c_p(T_{03} - T_{06})$$

$$= 1.005(1173 - 737.3) = 437.88\,\text{kJ/kg}$$

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Solution: Problem # 1

Since w_{net} remains the same,

$$\eta = \frac{w_{net}}{Q_1} = \frac{123.62}{437.9} = 0.2837 \; or \; 28.37\%$$

The percentage increase due to regeneration :

$$= \frac{0.2837\text{-}0.199}{0.199} = 0.4256 \; or \; 42.56\%$$

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Problem # 2

- A gas turbine operating at a pressure ratio of 11.314 produces zero net work output when 473.35 kJ of heat is added per kg of air. If the inlet air temperature is 300 K and the turbine efficiency if 71%, find the compressor efficiency.

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Solution: Problem # 2

- Since the net work output is zero,

$$w_c = w_t$$

$$\text{or, } T_{02} - T_{01} = T_{03} - T_{04}$$

$$T_{03} - T_{02} = T_{04} - T_{01}$$

$$\frac{T_{02s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma} = 11.314^{0.4/1.4}$$

$$T_{02s} = 300 \times 11.314^{0.4/1.4} = 600K$$

$$\text{Given that heat added} = 476.35 \text{ kJ/kg}$$

$$c_p (T_{03} - T_{02}) = 476.354$$

$$or, T_{03} - T_{02} = 474K$$

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Solution: Problem # 2

We know that

$$T_{04} = T_{01} + (T_{03} - T_{02}) = 300 + 474 = 774K$$

The turbine efficiency is 71%

$$0.71 = \frac{T_{03}(1 - T_{04}/T_{03})}{T_{04s}(T_{03}/T_{04s} - 1)} \text{ and } T_{03}/T_{04s} = 11.314^{0.4/1.4}$$

$$\therefore \frac{T_{04}}{T_{03}} = 1 - \frac{0.71}{2} = 0.645$$

$$or, T_{03} = 774/0.645 = 1200K \text{ and } T_{02} = 1200 - 474 = 726K$$

$$\therefore \eta = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{600 - 300}{726 - 300} = 0.704 \; or \; 70.4\%$$

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Problem # 3

- An aircraft flies at a Mach number of 0.75 ingesting an airflow of 80 kg/s at an altitude where the ambient temperature and pressure are 222 K and 10 kPa, respectively. The inlet design is such that the Mach number at the entry to the inlet is 0.60 and that at the compressor face is 0.40. The inlet has an isentropic efficiency of 0.95. Find (a) the area of the inlet entry (b) the inlet pressure recovery (c) the compressor face diameter.

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Solution: Problem # 3

- Mach number is 0.75, hence, the flight speed is

$$u_a = M_a \sqrt{\gamma R T_a} = 0.75\sqrt{1.4 \times 287 \times 222} = 224\ m/s$$

$$\rho_a = P_a / R T_a = 0.1569\ kg/m^3$$

$$\text{The total temperature, } T_{0a} = T_a \left(1 + \frac{\gamma - 1}{2} M_a^2\right) = 246\,K$$

$$\text{Total pressure, } P_{0a} = P_a \left(1 + \frac{\gamma - 1}{2} M_a^2\right)^{\gamma/(\gamma-1)} = 14522.8\ Pa$$

$\therefore$ Static temperature at inlet entry,

$$T_1 = T_{0a} / \left(1 + \frac{\gamma - 1}{2} M_a^2\right) = 230.4\ K$$

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Solution: Problem # 3

Static pressure at inlet entry,

$$P_1 = P_{0a} \Big/ \left(1 + \frac{\gamma - 1}{2} M_a^2\right)^{\gamma/(\gamma-1)} = 11386\ Pa$$

$$\rho_1 = P_1 / RT_1 = 0.1722\ kg/m^3$$

Therefore, area at the inlet entry, $A_1 = \dfrac{\dot{m}}{u_1 \rho_1} = \dfrac{\dot{m}}{M_1 \sqrt{\gamma R T_1}\, \rho_1}$

$$= \frac{80}{0.6\sqrt{1.4 \times 287 \times 230.4} \times 0.1722}$$

$$= 2.54\ m^2$$

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Solution: Problem # 3

Now, $T_{02} = T_{0a}$

Diffuser efficiency, $\eta_d = \dfrac{T_{02s} - T_a}{T_{0a} - T_a}$

Substituting the values, $T_{02s} = 245.75K$

$\therefore$ Pressure recovery, $\dfrac{P_{02}}{P_{01}} = \left(\dfrac{T_{02s}}{T_{01}}\right)^{\gamma/(\gamma-1)} = 0.982$

The static pressure at the compressor face, T_2

$$T_2 = T_{02} / \left[1 + \frac{\gamma - 1}{2} M_2^2\right] = 239.3\,K$$

$$P_2 = P_1 \left(T_2 / T_1\right)^{\gamma/(\gamma-1)} = 13001\ Pa$$

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Solution: Problem # 3

$$\rho_2 = P_2 / RT_2 = 0.1893\, kg/m^3$$

Velocity at the compressor face, $u_2 = M_2 \sqrt{\gamma R T_2}$

$$= 124.03\ m/s$$

Area of the compressor face, $A_2 = \dot{m} / u_2 \rho_2$

$$= 3.407\, m^2$$

$\therefore$ the diameter, $\mathrm{d} = 2.08\,\mathrm{m}$

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Problem # 4

- A turbojet engine operates at an altitude where the ambient temperature and pressure are 216.7 K and 24.444 kPa, respectively. The flight mach number is 0.9 and the inlet conditions to the convergent nozzle are 1000 K and 60 kPa. If the nozzle efficiency is 0.98, the ratio of specific heat is 1.33, determine whether the nozzle is operating under choked condition or not. Determine the nozzle exit pressure.

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Solution: Problem # 4

The nozzle efficiency is defined as

$$\eta_n = \frac{h_{06} - h_7}{h_{06} - h_{7s}} = \frac{T_{06} - T_7}{T_{06} - T_{7s}} = \frac{1 - T_7 / T_{06}}{1 - T_{7s} / T_{06}} = \frac{1 - T_7 / T_{07}}{1 - T_{7s} / T_{06}}$$

Under choked condition, $\mathrm{M} = 1$,

$$\therefore \eta_n = \frac{1 - (2/(\gamma + 1))}{1 - (P_c / P_{06})^{(\gamma - 1)/\gamma}}$$

$$or, \frac{P_{06}}{P_c} = \frac{1}{(1 - (1/\eta_n)((\gamma - 1)/(\gamma + 1)))^{\gamma/(\gamma - 1)}}$$

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Solution: Problem # 4

Substituting the values,

$$\frac{P_{06}}{P_c} = 1.878$$

Also, $\dfrac{P_{06}}{P_a} = \dfrac{60}{24.444} = 2.45Pa$

We can see that $P_c > P_a$

Therefore, the nozzle is operating under choked condition.

The exit pressure would therefore be equal to

$$P_e = \frac{60}{1.878} = 31.95kPa$$

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Exercise Problem # 1

- A Brayton cycle with two stages of compression and two stages of expansion has an overall pressure ratio of 8.0. Air enters each stage of the compressor at 300 K and each stage of the turbine at 1300 K. Determine the thermal efficiency (a) with no regenerator (b) with an ideal regenerator (c) if compressor and turbine have 80% efficiency, no regenerator.
- Ans: (a) 35%, (b) 69.6 %, (c) 26 %

Exercise Problem # 2

- In a gas turbine plant, the air at the inlet is at 27^{o}C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800^{o}C. The turbine and compressor efficiencies are 80% each. Find (a) the compressor work per kg of air (b) the turbine work per kg of air (c) heat supplied per kg of air and (d) the cycle efficiency.
- Ans: (a) 259.4 kJ/kg, (b) 351.68 kJ/kg (c) 569.43 kJ/kg (d)16.2 %

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Exercise Problem # 3

- An aircraft is flying at a Mach number of 0.8 at an altitude where the ambient static pressure is 40 kPa. If the diffuser pressure recovery is 0.9, determine the isentropic efficiency of the diffuser.
- Ans: 0.738

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Exercise Problem # 4

- The nozzle of a turbojet engine develops a thrust of 590 Ns/kg. The aircraft is flying at 240 m/s. The pressure and temperature at the nozzle entry are 1.284 kPa and 993 K, respectively. If the ratio of specific heat is 0.33, determine the nozzle efficiency. The nozzle can be assumed to be operating under choked condition.
- Ans: 0.95

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