Introduction to Aerospace Propulsion

Prof. Bhaskar Roy, Prof. A M Pradeep
Department of Aerospace Engineering, IIT Bombay

Lecture No - 21

In this lecture …

- Properties of pure substances
  - Compressed liquid, saturated liquid, saturated vapour, superheated vapour
  - Saturation temperature and pressure
  - Property diagrams of pure substances
  - Property tables
  - Composition of a gas mixture
  - P-v-T behaviour of gas mixtures
  - Ideal gas and real gas mixtures
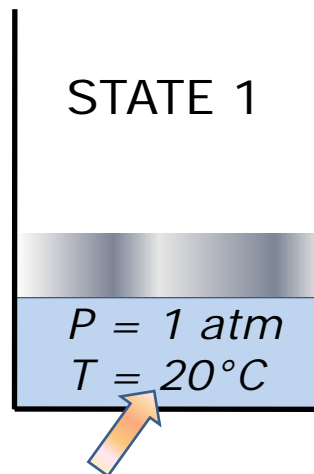 - Properties of gas mixtures

Properties of pure substances

- A substance that has a fixed chemical composition throughout is called a pure substance. Eg. Water, nitrogen, helium etc.

- A mixture of two or more phases of a pure substance is still a pure substance as long as the chemical composition of all phases is the same. Eg. Mixture of ice and water.

- Properties of water (and its different phases) important part of analysis.
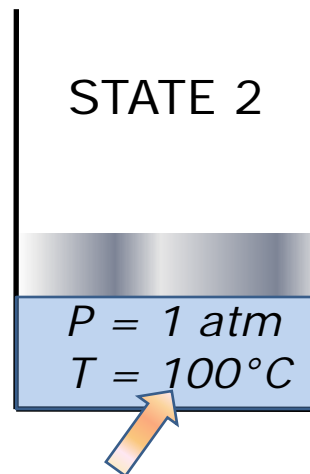
Properties of pure substances

- When water (or any other liquid) exists in a state when it is not about to vapourize: compressed liquid or subcooled liquid. Eg. water at 20°C, 1atm.

- A liquid that is about to vaporize is called a saturated liquid.

- A vapour that is about to condense is called a saturated vapour.

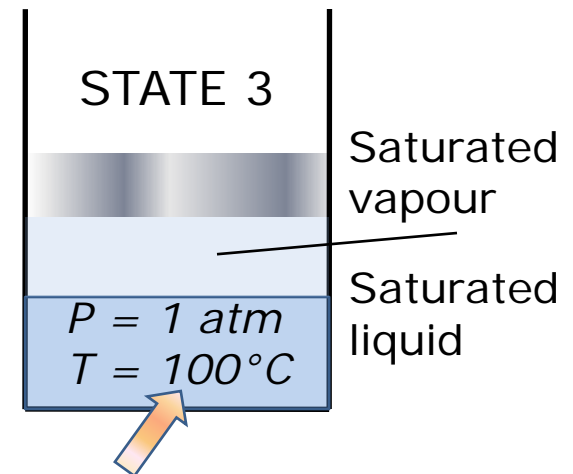- A vapour that is not about to condense (i.e., not a saturated vapour) is called a superheated vapour.

Properties of pure substances

STATE 1

P = 1 atm
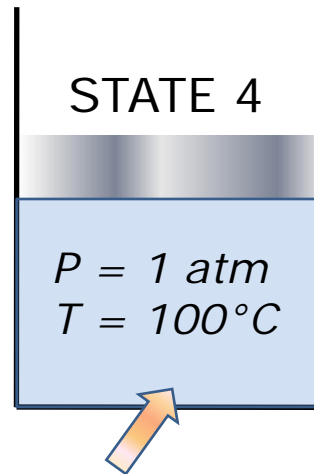T = 20°C

Compressed liquid

STATE 2

P = 1 atm
T = 100°C

Saturated liquid

STATE 3

Saturated vapour

Saturated liquid

P = 1 atm
T = 100°C

Saturated liquid–vapour mixture

Different states of water

Properties of pure substances

STATE 5

STATE 4

$P = 1\ atm$
$T = 100°C$
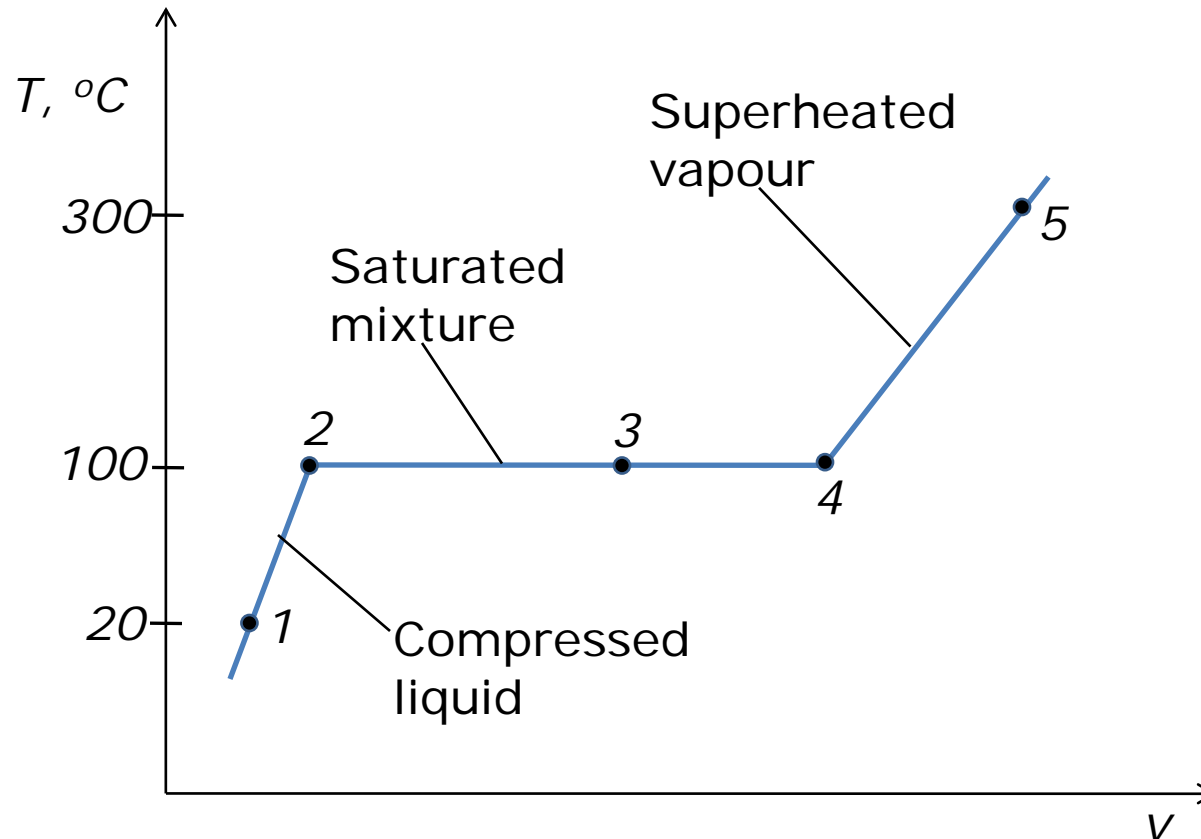
$P = 1\ atm$
$T = 300°C$
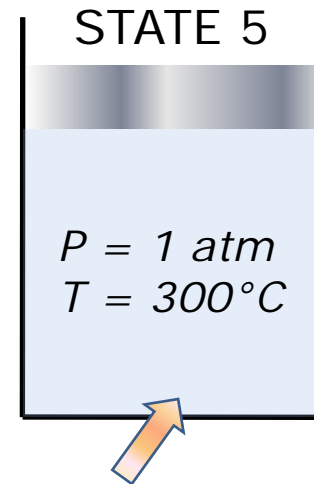
Saturated vapour   Superheated vapour

Different states of water

Properties of pure substances

T-v diagram for the heating process of water at constant pressure (P=1atm.)

Properties of pure substances

- The temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature.

- At a given pressure, the temperature at which a pure substance changes phase is called the saturation temperature, T_{sat}.

- Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the saturation pressure, P_{sat}.

Properties of pure substances
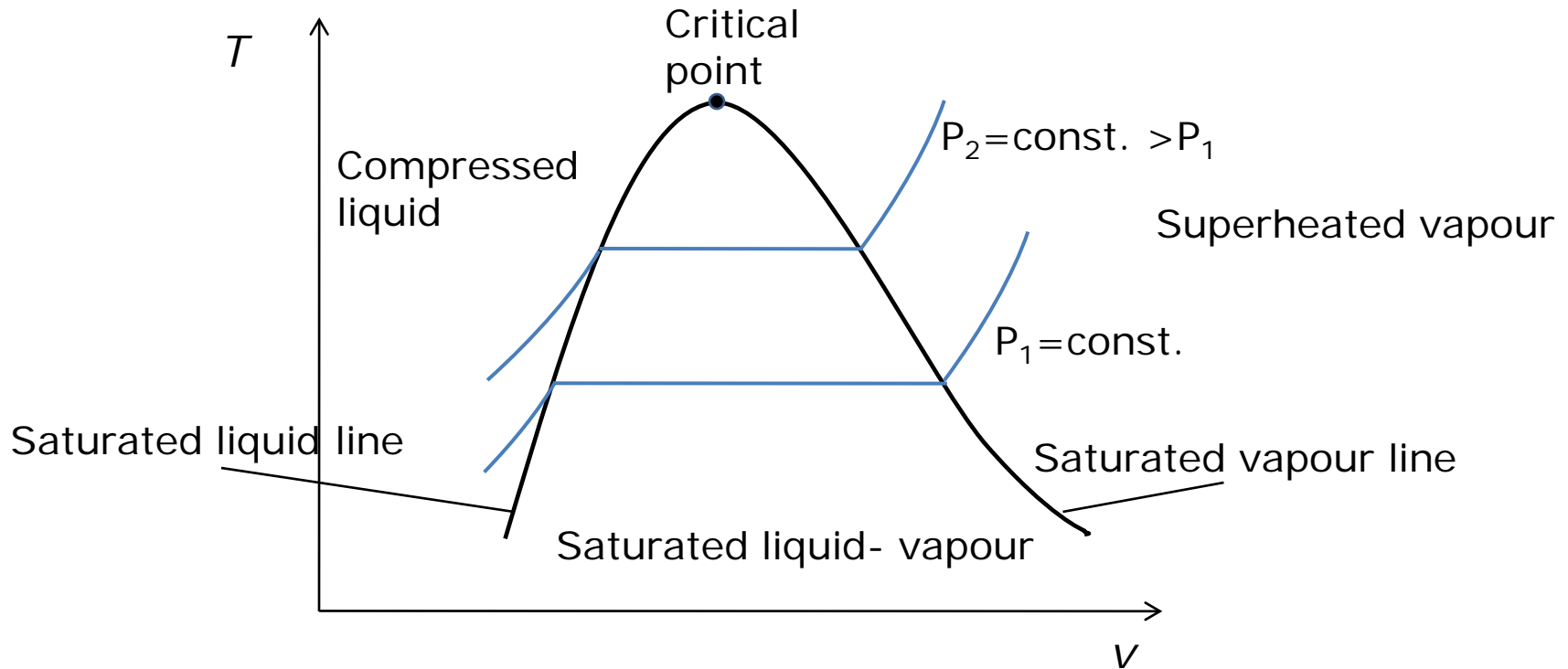
T-v diagram of a pure substance

Properties of pure substances
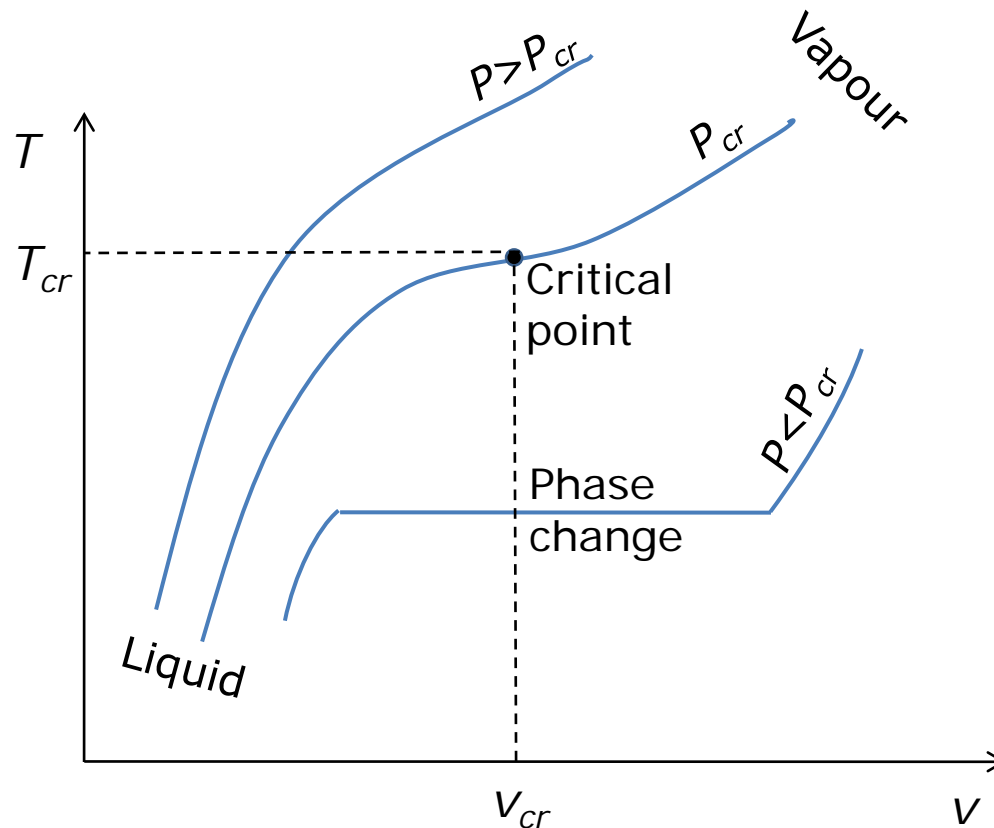
At supercritical pressures $(P > P_{cr})$, there is no distinct phase-change (boiling) process.

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Properties of pure substances
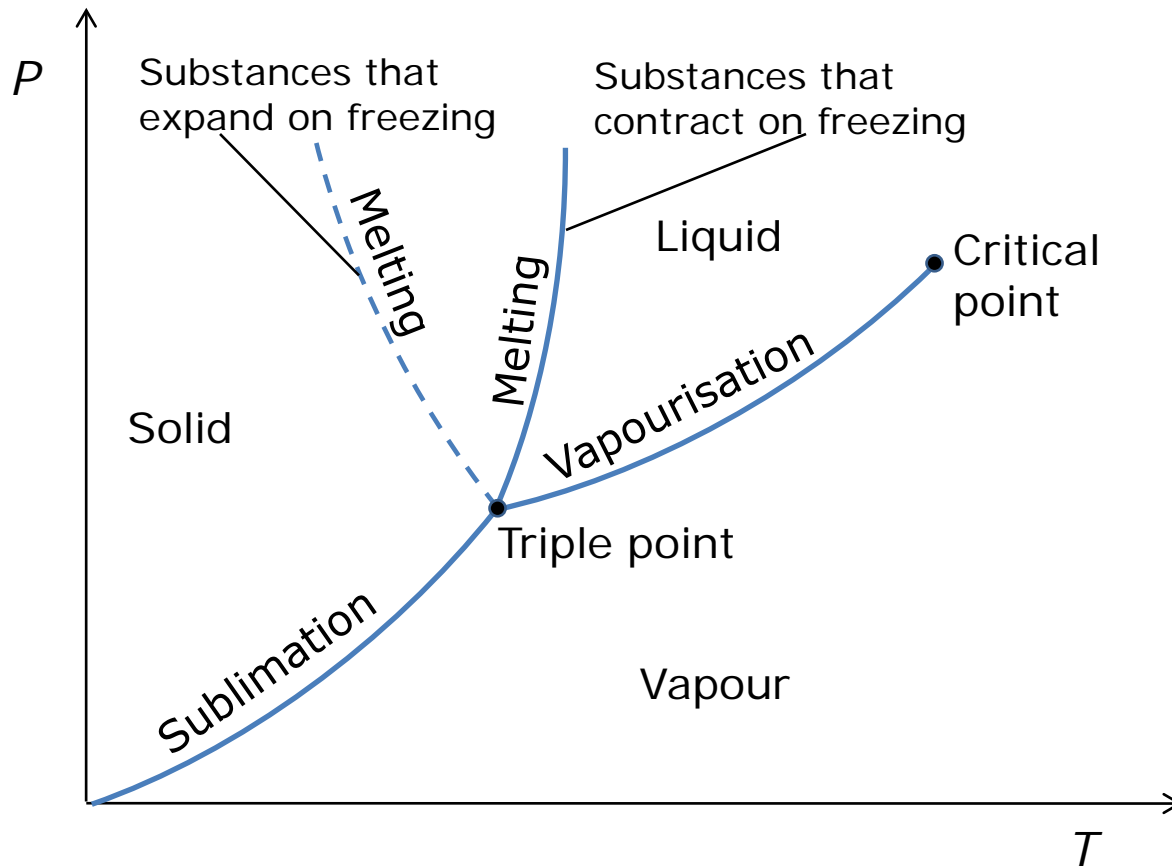
P-T diagram of pure substances

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Property tables

- The relationships among thermodynamic properties are too complex to be expressed by simple equations.

- Properties are frequently presented in the form of tables.

- The subscript, *f,* denotes properties of a saturated liquid, and the subscript, *g,* denotes the properties of saturated vapour.

- *fg*, denotes the difference between the saturated vapour and saturated liquid values of the same property.

Property tables

- For eg.:

h_f=specific enthalpy of saturated liquid

h_g=specific enthalpy of saturated vapour

h_{fg}=h_g−h_f Enthalpy of vapourisation or

latent heat of vapourisation

- Quality, *x*, is defined as the ratio of the mass of vapour to the total mass.

- Quality has a value ranging between 0 and 1.

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Property tables

- $x=0 \rightarrow$ saturated liquid, $x=1 \rightarrow$ saturated vapour

- It can be shown that in general,

$$y_{avg} = y_f + x\, y_{fg}$$

  where, y can be: *v, u, s* or *h*

- For eg: $h_{avg} = h_f + x\, h_{fg}$

- Usually, the subscript *avg* is dropped for simplicity.

- Also, $y_f \leq y_{avg} \leq y_g$

Superheated vapour

- Region to the right of the saturated vapour line and at temperatures above the critical point temperature.

- In the superheated region (single phase), pressure and temperature are no longer dependant properties.

- Compared to saturated vapour, superheated vapour is characterised by:
  - Lower pressures ($P < P_{sat}$ at a given T)
  - Higher temperatures ($T > T_{sat}$ at a given P)
  - Higher specific volume/enthalpy/internal energy ($v > v_g$, $h > h_g$, $u > u_g$ at a given P or T)

Properties of pure substances
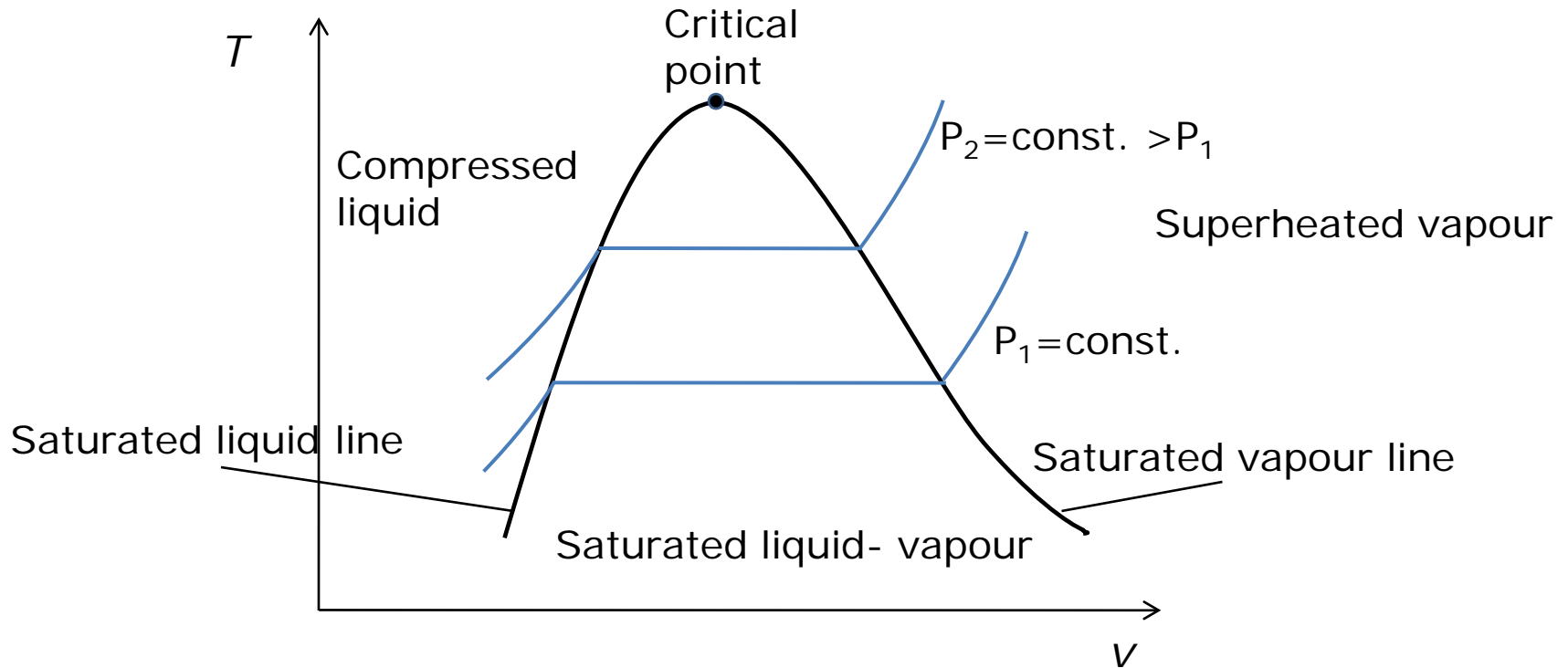
T-v diagram of a pure substance

Composition of a gas mixture

- Consider a gas mixture of *k* components.
- Let m_m is the mass of the mixture and N_m is sum of the mole numbers.

$$m_m = \sum_{i=1}^{k} m_i \quad \text{and} \quad N_m = \sum_{i=1}^{k} N_i$$

- Mass fraction, *mf,* is the ratio of the mass of a component to the mass of the mixture
- Mole fraction, *Y,* is the ratio of the mole number of a component to the mole number of the mixture.

$$mf_i = m_i / m_m \quad \text{and} \quad y_i = N_i / N_m$$

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Composition of a gas mixture

- The mass of a substance of mole number N and molar mass M is $m = NM$

- The average molar mass and gas constant:

$$M_m = \frac{m_m}{N_m} = \frac{\sum m_i}{N_m} = \frac{\sum N_i M_i}{N_m} = \sum_{i=1}^{k} y_i M_i$$

$$R_m = \frac{R_u}{M_m}$$

- Mass and mole fractions of a mixture are related by
$$mf_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

P-v-T behaviour of gas mixtures

- Ideal gas equation of state with compressibility factor for real gases.

- The prediction of the P-v-T behaviour of gas mixtures based on two laws: Dalton's law of additive pressures and the Amagat's law of additive volumes.

- Dalton's law of additive pressures: The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume.

P-v-T behaviour of gas mixtures

- Amagat's law of additive volumes: The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.

- Dalton's and Amagat's laws hold exactly for ideal-gas mixtures, but only approximately for real-gas mixtures.

- For ideal gases, these two laws are identical and give identical results.

P-v-T behaviour of gas mixtures

- Amagat's law of additive volumes: The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.

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P-v-T behaviour of gas mixtures

- Dalton's and Amagat's laws can be expressed as follows:

$$\text{Dalton's law}: \quad P_m = \sum_{i=1}^{k} P_i(T_m, V_m)$$

$$\text{Amagat's law}: \quad V_m = \sum_{i=1}^{k} V_i(T_m, V_m)$$

- The above are exact for ideal gases, but approximate for real gases.

- P_i is called the component pressure and V_i is called the component volume.

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Ideal gas mixtures

- For ideal gases, P_i and V_i can be related to the mole fraction y_i by:

$$\frac{P_i(T_m, V_m)}{P_m} = \frac{N_i R_u T_m / V_m}{N_m R_u T_m / V_m} = \frac{N_i}{N_m} = y_i$$

$$\frac{V_i(T_m, V_m)}{V_m} = \frac{N_i R_u T_m / V_m}{N_m R_u T_m / V_m} = \frac{N_i}{N_m} = y_i$$

Therefore, $\dfrac{P_i}{P_m} = \dfrac{V_i}{V_m} = \dfrac{N_i}{N_m} = y_i$

Ideal gas mixtures

- The quantity $y_i P_m$ is called the partial pressure and the quantity $y_i V_m$ is called the partial volume.

- For an ideal-gas mixture, the mole fraction, the pressure fraction, and the volume fraction of a component are identical.

Real gas mixtures

- Dalton's law of additive pressures and Amagat's law of additive volumes can also be used for real gases, with some approximations.

- We either use some of the advanced equations of state (like the Beattie–Bridgeman, Benedict–Webb–Rubin etc.)

- The other way is to use the compressibility factor: $PV = ZNR_uT$

Real gas mixtures

- The compressibility factor of the mixture, Z_m, can be expressed in terms of that of the constituents:

$$Z_m = \sum_{i=1}^{k} y_i Z_i$$

- Here, Z_i can be determined either at T_m and V_m (Dalton's law) or at T_m and P_m (Amagat's law) for each individual gas.

- It is however not necessary that both these methods give the same result.

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Properties of gas mixtures

- To evaluate the extensive properties of a non-reacting ideal or real gas mixture, we add the contributions of each component.

- The internal energy, enthalpy and entropy can be expressed as:

$$U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i$$

$$H_m = \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i$$

$$S_m = \sum_{i=1}^{k} S_i = \sum_{i=1}^{k} m_i s_i$$

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Properties of gas mixtures

- The internal energy, enthalpy, and entropy of a mixture *per unit mass of the mixture can be determined by dividing the equations* above by the mass of the mixture:

$$u_m = \sum_{i=1}^{k} mf_i u_i \quad \text{and} \quad h_m = \sum_{i=1}^{k} mf_i h_i \quad \text{in (kJ/kg)}$$

$$s_m = \sum_{i=1}^{k} mf_i s_i \quad \text{(kJ/kg.K)}$$

$$\text{Also, } c_{v,m} = \sum_{i=1}^{k} mf_i c_{v,i} \quad \text{(kJ/kg.K) and } c_{p,m} = \sum_{i=1}^{k} mf_i c_{p,i}$$

Properties of gas mixtures

- Similarly, the changes in internal energy, enthalpy, and entropy of a gas mixture during a process can be expressed:

$$\Delta U_m = \sum_{i=1}^{k} \Delta U_i = \sum_{i=1}^{k} m_i \Delta u_i$$

$$\Delta H_m = \sum_{i=1}^{k} \Delta H_i = \sum_{i=1}^{k} m_i \Delta h_i$$

$$\Delta S_m = \sum_{i=1}^{k} \Delta S_i = \sum_{i=1}^{k} m_i \Delta s_i$$

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  - Properties of gas mixtures

In the next lecture …

- One dimensional compressible flows
- Stagnation properties
- Speed of sound and Mach number
- One-dimensional isentropic flow
- Variation of fluid velocity with flow area
- Isentropic flow through nozzles
  - Converging nozzles
  - Converging–diverging nozzles

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